Earnings manipulation in an information choice environment ⋆

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Abstract

I study earnings manipulation in a rational expectations equilibrium model with moral hazard and stock-based compensation. Managers are able to bias the financial statement, while speculators observe financial statements and acquire private information about the companies' payoffs. I show three main results. First, environments with smaller information asymmetry between managers and market participants provide stronger incentives for managers to manipulate. Second, at equilibrium, the intensity of performance pay is smaller for managers working in environments with smaller information asymmetry, counteracting (and overtaking) the incentives coming from asymmetric information itself. Third, I consider an environment where a regulator more intensively scrutinizes companies with higher information asymmetry. In that case, managers who bias the financial statement the most face only intermediate detection probabilities and intermediate values of performance pay at equilibrium.

Keywords: Earnings manipulation, Financial reporting, Information environment.
JEL classification: D82, G14, M42, M48.

1. Introduction

Many theoretical studies have considered earnings manipulation as an attempt of managers to increase their pay. Two different perspectives have been adopted in the literature. The first strand consists of models with optimal contracting when the manager's report is the only mutually observed variable (e.g., Beyer et al., 2012; Sun, 2014). Models on the second strand explicitly restrict the functional form of the contract while letting it depend on variables linked to the report, such as the stock price (e.g., Goldman and Slezak, 2006; Peng and Röell, 2014). Common to both of them is the underlying assumption that the manager's actual or perceived ability to affect his pay through manipulation is the same for all managers or, if different across managers, is exogenously given.

I present a model in which the manager's perceived ability to affect his pay through manipulation is potentially different across managers and is endogenously determined in equilibrium. Building on the second strand, the manager's perceived ability to boost the stock price depends on the extent to which market participants rely on the manager's report when forming their expectations. The higher the manager's report weight on their expectation, the more this report is reflected on prices, and the greater the manager's perceived ability to boost prices. In an environment with information acquisition, the extent to which market participants' expectations depend on the report is endogenous. For instance, if market participants choose not to acquire any private information on a given payoff, then the manager's report is all they have available to form their expectations. In this case, the stock price reflects much of the manager's report, which provides the manager with a great perceived ability to boost prices.

In my model, managers' earning reports constitute potentially biased estimates of payoffs. A more precise report is one that provides a more precise estimate of the company's current state. The precision of the report ultimately reflects the degree of information asymmetry between the manager and market participants: the more precise the report, the smaller the information asymmetry. I

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show that environments with smaller information asymmetry between managers and market participants[2] provide stronger incentives for the manager to manipulate. The reason is as follows. In equilibrium, market participants acquire on average less information on payoffs about which they are initially more informed, i.e., those associated to reports that are more precise. Because price aggregates private information and partially reveals it, more precise reports will have more weight on stock prices. Thus, the more precise the financial statement, the more the manager perceives himself capable of affecting the stock price through manipulation; for a given contract, he manipulates to a larger extent.

The entrepreneur who designs the contract takes into account the incentives coming from the information environment. In companies with smaller information asymmetry between the manager and market participants, he chooses lower performance pay to diminish the manager's incentive to manipulate. In my model, the impact on manipulation coming from compensation is stronger: in companies with smaller information asymmetry, both performance pay and financial statement bias are lower.

Earnings manipulation is not easy to detect because earnings are not directly and objectively possible to observe. Furthermore, the Generally Accepted Accounting Principles (GAAP) allow for some managerial discretion in assessing earnings. The GAAP revenue recognition principle is accrual-based. Companies must record revenue when it is earned but not when received; losses must be recorded when their occurrence becomes probable, whether or not it has actually occurred. Consider the following example. Suppose that a company purchases a finite-stochastically lived machine that produces a good that, when sold, generates revenue for the company. If buyers purchase the good on credit, then the value recorded must take into account the probability of default. Thus the recorded value may be higher or lower depending on the manager's discretion in assessing such probability.

Also, the GAAP matching principle establishes that expenses must be matched with revenues. Thus the cost incurred by the company when it purchases the machine must be distributed over the period of the machine's service life. The fact that the machine's service life is stochastic constitutes another instance in

[2] Beyer et al. (2014) also consider a setting in which the fundamental economic uncertainty is distinguished from the information asymmetry between the manager and market participants. However, they abstract from the contracting problem and its endogenous interaction with the financial statement informativeness in providing incentives for the manager to manipulate. For a review of the financial reporting environment literature, see Beyer et al. (2010).

which the manager can exercise his discretion. Finally, as time goes by, earnings reports must also account for the depreciation of the machine which is not directly observable as well.

There is a wide literature in accounting providing evidence on both the possibility and the incentives to manipulate earnings (Ronen and Yaari, 1981; Merchant, 1989; Buckmaster, 2001). Empirical studies find evidence that managers whose compensation is more linked to stock prices are more likely to engage in earnings management and manipulation (Bergstresser and Philippon, 2006; Burns and Kedia, 2006)[3]. Anecdotal evidence confirms the empirical results, suggesting that one of the main reasons for earnings manipulation is to influence the stock price. Dichev et al. (2013) survey 169 CFOs of public companies and report that 93.45% of them agree with the assertion that companies misrepresent earnings to influence the stock price, the highest agreement rate among all the possible motivations presented.

In the model that I present, a manager has incentives to manipulate the financial statement because he believes that, by doing so, he can raise stock prices which is part of his compensation package. The model shows how the manager's decision to manipulate and its extent depend on the weight that speculators place on the financial statement. For instance, if those who trade claims on the company's payoff completely disregard the financial statement, then the manager could not possibly conceive manipulation as an attempt to boost stock price; at the other extreme, if the financial statement is the only source of information that can be used to predict the payoff, then the incentive to manipulate through this channel will be fully in place. In general, the financial statement is but one of the sources of information that speculators have available to build their expectations.

I introduce information acquisition to an otherwise standard model of moral hazard and stock-based compensation. Managers are hired by entrepreneurs (insiders) to operate a technology and are offered a stock-based contract. Each manager then undertakes unobservable effort and, before the final payoff is realized,

[3]There is no consensus in the literature on the definition of earnings management. Ronen and Yaari (2008) survey the different definitions and propose the following: "earnings management is a collection of managerial decisions that result in not reporting the true short-term, value-maximizing earnings as known to management." Their definition is flexible enough to admit the possibility of earnings management being beneficial. This would be the case if the manager misrepresents the short run to better represent the long run which, in some circumstances, is what market participants really care about. In the model that I present here is no such trade-off because the better the financial statement represents the short run, the better it represents the long run as well.

observes the intermediate value (earnings) which they must report. Managers are able to bias the financial statement at a (resource) cost. The final payoff is given by the sum of the intermediate value and the realization of a shock, net of the (resource) cost of manipulation to the company.

Speculators acquire private information and combine it with the other information they have in order to form their payoff expectations, and then trade claims on the payoffs. The weight of the financial statement on speculators' expectation, which I call *financial statement informativeness*, depends on how precise the statement is as an estimate of the intermediate value.

Building information into the model allows me to consider the impact of some aspects of regulatory measures concerning the information structure. For instance, one of the elements of the Sarbanes-Oxley Act of 2002 (SOX), a reaction to the accounting scandals that paved the way for the stock market downturn of that year[4], promoted the enhancement of financial disclosures. This is captured by the financial statement precision in my model.

My results show that an increase in the report's precision can drive both the contract performance sensitivity and the bias in the report down. This result is consistent with the empirical evidence that the share of incentive-based compensation and earnings manipulation decreased after the SOX bill was passed (Cohen et al., 2007, 2008). In models like mine in which the contract performance sensitivity balances out desirable effort with undesirable manipulation, policy measures that increase the cost of manipulation to the manager can account for the decrease in reports bias but performance pay would necessarily increase. Information is one way to make models of this class conform to the empirical evidence.

I also present an extension of my model to study detection of manipulation. Specifically, I consider the case in which a regulator chooses the intensity at which reports are scrutinized. I also assume that public information is released as the scrutiny activity takes place. In this manner, there is an explicit link between public information and detection probability[5]. I show that, in equilibrium, companies with different financial statement informativeness would face different detection

[4] Notably at Enron and WorldCom.

[5] It could be the case that the more the company is under public scrutiny (measured by the precision of the public information about its payoff), the more insiders might attempt to hide the manipulation, making it more difficult to be detected. Although this seems to be a theoretical possibility it is not a compelling one. Other insiders have access to the company's (true) financial information and suffer the consequences of manipulation, while the benefit might not be as high as it is for the manager. Thus, those other insiders would an incentive to blow the whistle (see Friebel and Guriev, 2012).

probabilities, and I characterize how the report's bias and the contract performance sensitivity would respond to changes in report's precision.

I solve for the case in which the regulator scrutinizes at a higher intensity companies with higher information asymmetry between managers and market participants. This policy delivers the most favorable scrutiny distribution to speculators and can be thought as being the outcome of speculators' calls for SEC scrutiny of some companies. While they are usually thought as being associated to short selling, my model provides yet another interpretation, one based on information. Since scrutiny results in the release of public information, speculators have a preference over which payoffs are scrutinized because of the uncertainty resolution that follows. I find that, in equilibrium, managers biasing the reports the most are not the ones issuing either the most or the least informative financial statements. As for the contract performance sensitivity, in some circumstances, it is the highest for the managers issuing the least informative statements. Furthermore, managers who are most likely to get caught are not those biasing the most but, on the contrary, they are some of those biasing the least in equilibrium.

This result is relevant for the design of policies attempting to curb manipulation. It suggests that a policy that prescribes higher-intensity scrutiny at companies with higher information asymmetry between managers and market participants may not achieve the intended result and may not be optimal (it maximizes speculators' utility, but not necessarily social welfare).

2. Related literature

I study a noisy rational expectations equilibrium model in which speculators can acquire information about payoffs affected by a manager who can manipulate financial information in a contract setting with moral hazard. Thus this paper builds on the literature on noisy rational expectations (e.g., Admati, 1985), moral hazard (e.g, Mirrlees, 1971; Holmström, 1979; Grossman and Hart, 1983), information acquisition (e.g., Hellwig, 1980; Verrecchia, 1982; Grossman and Stiglitz, 1980; Van Nieuwerburgh and Veldkamp, 2010), manipulation of financial statements (e.g., Baiman et al., 1987; Dye, 1988; Demski, 1998). It also contributes to the literature that studies the feedback effect from financial markets to the real economy due to the informational role of prices[6] (e.g., Kanodia and Lee, 1998; Goldstein and Guembel, 2008).

[6] See Bond et al. (2012) for a review of the feedback effects literature.

This paper belongs to the literature on implications of manipulation on contracting. Like Dutta and Gigler (2002), Liang (2004), Goldman and Slezak (2006), Sun (2014) and Peng and Röell (2014), I consider a risk averse agent[7]. Dutta and Gigler (2002) consider a model in which the manager reports earnings and also issues an earnings forecast. In their mechanism design, the manager's earnings forecast is reported truthfully (Revelation Principle); still, they show that basing the contract also on potentially manipulated earnings reduces the informational rent the manager extracts. Liang (2004) restricts attention to linear contracts in a two-period model. He shows that a policy of zero-tolerance to earnings management may be undesirable because some earnings management reduces agency costs.

Like Goldman and Slezak (2006), I present a model restricted to linear contracts; the manager chooses effort and may also spend the company's resources by manipulation in an attempt to boost the price of the security in which his compensation is based. I build on their model by explicitly modeling the speculators' expectations and decisions based on the different pieces of information available.

Sun (2014) studies a model in which whether a manager can or cannot misreport is stochastically determined. Thus the Revelation Principle does not hold and the principal does not know whether misreporting has occurred in equilibrium. In her model, misreporting works as insurance to the manager when the payoff is low, weakening thus the incentives for (costly) effort exertion. As a result, the possibility of misreporting actually requires a higher performance pay. Unlike her model, in mine the contract performance sensitivity balances out effort and manipulation.

In Peng and Röell (2014) manipulation propensity is stochastic and realized after the contract is signed. Thus the amount of manipulation is not perfectly anticipated in equilibrium. In their setting, the higher the manipulation uncertainty, the smaller the price sensitiveness to the report which, in turn, leads to smaller amount of manipulation. Unlike their model, in mine the amount of manipulation is perfectly known in equilibrium and the (perceived) price sensitiveness to the report depends on how much information is conveyed through the financial statement.

[7] Some studies featuring a risk-neutral agent that can manipulate the performance measure are Maggi and Rodríguez-Clare (1995), Crocker and Morgan (1998), Fischer and Verrecchia (2000), Crocker and Slemrod (2007) and Beyer et al. (2012).

3. The model

Time is discrete and there are four periods, $t \in \{0,1,2,3\}$. There exist n types of technologies; each type $j \in \{1,\dots,n\}$ generates a (gross) payoff π_j in the last period. The payoff increases with effort exerted e_j, decreases with manipulation (report's bias) m_j and depends on the shocks $\chi_j \sim \mathcal{N}\left(0, V_{\chi j}\right)$ and $\epsilon_j = \bar{\epsilon}_j + \xi_{\epsilon j}$ with $\xi_{\epsilon j} \sim \mathcal{N}\left(0, \Sigma_{\epsilon j}\right)$. The magnitude of the impact of manipulation on the payoff is weighted by the incremental resource cost $\alpha \in (0,1)$, which I interpret as being the opportunity cost in terms of payoff of having a manager dedicating his time and the company's resources to such endeavor[8]. Formally:

$$\pi_j = \beta_j e_j + \chi_j - \alpha_j m_j + \epsilon_j$$

where β_j is the payoff sensitivity to effort[9].

An *intermediate value* (earnings) $\pi_{1j} \equiv \beta_j e_j + \chi_j$ is privately revealed to the manager operating the technology at $t = 1$, and the payoff π_j is revealed to everybody at the final period. Although the intermediate value is not publicly observed, an auditor can observe it with noise, i.e., $\beta_j e_j + \chi_j - \xi_{\theta j}$, where $\xi_{\theta j} \sim \mathcal{N}\left(0, \Sigma_j\right)$, and release a *financial statement* θ_j which would then be publicly available. This statement would be a noisy observation of the intermediate value. However, the manager can take actions such as misrepresenting information to distort the mean of the auditor's assessment by m_j (report's bias). Thus $\theta_j = \beta_j e_j + \chi_j + m_j - \xi_{\theta j}$.

The variance of the assessment represents the information asymmetry between the manager and the market participants who ultimately will make use of the auditor's report. I assume that $\Sigma_{\epsilon j} > \Sigma_j$ for every payoff j, that is, the intratemporal uncertainty on the noisy observation is never greater than the intertemporal uncertainty. I assume further that $\frac{1}{n}\sum_l \left(\Sigma_l + \Sigma_{\epsilon l}\right)^{-1} < \Sigma_{\epsilon j}^{-1}$ for every payoff j.

There exist n risky assets, each of them corresponding to a claim on the *net payoff* associated to a technology (net of payments made to the manager operating the technology). There is one risk-free asset paying a constant amount r.

[8] There is empirical evidence that manipulation is accompanied by distorted hiring and investment decisions (see Kedia and Philippon, 2009)

[9] In this paper I restrict attention to positive report's bias, that is, the upward bias of the financial statement. Overstatements are more prevalent in the data; for instance, using data from GAO (2002) and COMPUSTAT, Kedia and Philippon (2009) report that 80% of the restatements are negative as measured by revisions to reported net income.

There exist n types of entrepreneurs (insiders), a continuum $[0,1]$ of each type; each entrepreneur is endowed with a technology which he cannot operate and has to hire a manager at $t = 0$. His objective is to maximize the value of his revenue.

There exists a measure n of ex-ante identical managers able to operate the technology owned by the entrepreneurs. Each manager is paid to accomplish this task by exerting effort. Effort decision is made between $t = 0$ and $t = 1$ and the report's bias is chosen at $t = 1$ after the manager observes the realization of χ_j. His actions are chosen to maximize his utility function, which depends on his compensation T and the cost $c(e,m)$, which is a function of effort e and the report's bias m:

$$\max_{e\geq 0, m\geq 0} \mathbb{E}[T] - \frac{\gamma}{2}\mathbb{V}[T] - c(e,m)$$

where γ is the risk aversion. I assume that $c(e,m) = \frac{c_e}{2}e^2 + c_m(m)$, where the c_e is a parameter for the cost of effort and c_m is a function of the report's bias m. I assume further that manipulation is privately costless for the manager if not for the possibility of being caught and fined. Thus I treat the fine in the same way as the compensation, that is, by means of a mean-variance utility. Letting k denote the probability that manipulation will be detected and g denote the fine per unit of bias, I define G as a random variable that takes value g with probability k and value 0 with probability $1-k$. Thus:

$$c_m(m) = \mathbb{E}[mG] + \frac{\gamma}{2}\mathbb{V}[mG] = kgm + \frac{\gamma}{2}k(1-k)g^2m^2$$

Managers and entrepreneurs are pairwise grouped only once at $t = 0$. I restrict attention to linear contracts comprised of a salary and stocks $T_j = h_{0j} + h_{1j}p_j$, where h_0 is the salary and h_1 represents shares to be transferred from the entrepreneur's account. The entrepreneur offers a contract that is accepted by the manager if it delivers an expected utility higher than his outside value Ω:

$$\mathbb{E}_0\left[T_j\right] - \frac{\gamma}{2}\mathbb{V}_0\left[T_j\right] - c_j\left(e_j, m_j\right) \geq \Omega$$

Claims on the payoffs are traded in a competitive market without any transaction cost at $t = 2$.

There is a continuum $[0,1]$ of ex-ante identical speculators, indexed by i, undertaking two decisions: at time $t = 1$ each speculator decides how much uncertainty to reduce about payoffs and at time $t = 2$ he chooses his asset position u_i to maximize a CARA utility function. His choice is constrained by his budget and the maximal reduction of uncertainty is capped by a constant (Van Nieuwerburgh and Veldkamp, 2010).

Speculators observe neither intermediate values, nor the report's bias in the financial statement, but they use the information contained in the financial statement and rely on a private and a public signal (released as a result of scrutiny promoted by a regulator) to make portfolio decisions. Specifically, they observe financial statements $\langle \theta_j \rangle_{j=1}^{n}$; once appropriately discounted by a quantity s_j, $\tilde{\theta}_j \equiv \theta_j - s_j + \bar{\epsilon}_j$ provides information about the payoff π_j:

$$\pi \sim \mathcal{N}\left(\tilde{\theta}, \tilde{\Sigma}\right), \quad \tilde{\Sigma} \equiv \begin{pmatrix} \tilde{\Sigma}_{1,1} & 0 & 0 & \cdots & 0 \\ 0 & \tilde{\Sigma}_{2,2} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \tilde{\Sigma}_{n,n} \end{pmatrix}, \quad \tilde{\Sigma}_{j,j} \equiv \mathbb{V}\left[\xi_{\theta j} + \epsilon_j\right]$$

At time $t = 1$, each speculator observes a public signal $w \sim \mathcal{N}\left(\pi, \tilde{\Sigma}_w\right)$ and chooses the precision $\tilde{\Sigma}_{\eta i}^{-1}$ of a private signal $\eta_i \sim \mathcal{N}\left(\pi, \tilde{\Sigma}_{\eta i}\right)$ that he receives between times $t = 1$ and $t = 2$, where:

$$\tilde{\Sigma}_w \equiv \begin{pmatrix} \tilde{\Sigma}_{w,1,1} & 0 & 0 & \cdots & 0 \\ 0 & \tilde{\Sigma}_{w,2,2} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \tilde{\Sigma}_{w,n,n} \end{pmatrix}$$

$$\tilde{\Sigma}_{\eta i} \equiv \begin{pmatrix} \tilde{\Sigma}_{\eta i,1,1} & 0 & 0 & \cdots & 0 \\ 0 & \tilde{\Sigma}_{\eta i,2,2} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \tilde{\Sigma}_{\eta i,n,n} \end{pmatrix}, \quad \int_0^1 \eta_i \, \mathrm{d}i = \pi$$

The assumption that the covariance matrix is diagonal means that signals associated to each payoff are independent.

This information choice problem is constrained by two inequalities: a no-forgetting constraint (NFC) and a capacity constraint. The former simply prevents the speculator from forgetting (or from deliberately not observing) the information publicly available. Mathematically:[10]

$$\tilde{\Sigma}_{\eta ij} \geq 0 \qquad \forall i, j \tag{NFC}$$

As for the latter, I assume an additive constraint (AC)[11]:

$$\sum_j \left[\hat{\Sigma}_{ij}^{-1} - \left(\tilde{\Sigma}_{ij}^{-1} + \tilde{\Sigma}_{wj}^{-1}\right)\right] \leq nK \qquad \forall i \tag{AC}$$

where $\hat{\Sigma}_i^{-1}$ is speculator's i posterior and nK is the capacity to be allocated for reducing uncertainty about the different payoffs. The assumed symmetry between speculators extends then to the uncertainty reduction capacity (K is the same across speculators). The posterior takes into account the fact that, by observing prices p, speculators may extract further information on the payoffs. Each speculator proceeds then, at time $t = 2$, to solve his portfolio problem:

$$\max_{u_i} \mathbb{E}_{2i}\left[\exp\left(-\rho W_i\right)\right]$$

s.t.

$$W_i = rW_{0i} + u_i'\left[\pi - (h_0 + H_1 rp) - rp\right] + \text{transfers}$$

where u_i is the speculator's portfolio, r is the risk-free return, ρ is the risk-aversion parameter, and W_i is the speculator's wealth (W_{0i} being initial wealth).

Let $(\mathbb{R}^n, \mathcal{B}(\mathbb{R}^n), \mathrm{p}_i)$ and F_i be the underlying probability space and the probability distribution function for the payoffs consistent with the belief, signal and conditional moments elicited from prices that speculator i uses to compute the moments $\mathbb{E}_i[W_i]$ and $\mathbb{V}_i[W_i]$, where $\pi - (h_0 + H_1 p)$ is the net payoff and

[10] Since all the matrices presented in this work are diagonal, I use the notation A_j for the term jj of matrix A.

[11] In Appendix E, I show that a version of the entropy learning technology would not change my results.

$$H_1 \equiv \begin{pmatrix} h_{1,1} & 0 & 0 & \cdots & 0 \\ 0 & h_{1,2} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & h_{1,n} \end{pmatrix}$$

To complete the model, there is a is a continuum $[0,1]$ of risk neutral liquidity traders, who purchase a share $1-\delta_j$ of the payoff from each technology j with the purpose of transferring value into the future and each is hit by a liquidity shock at time $t=2$ which cause him to increase or decrease his position. I let $y \sim \mathcal{N}\left(0, \sigma_y^2\right)$ be the shock hitting each liquidity trader which, in turn, translates into a shock to the asset supply[12]. Because the stake held by liquidity traders differs across payoffs, the shock to the asset supply may vary across assets and its distribution is given by:

$$x \sim \mathcal{N}\left(0, \Sigma_x^2\right), \qquad \Sigma_x^2 \equiv \begin{pmatrix} \sigma_{x1}^2 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_{x2}^2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \sigma_{xn}^2 \end{pmatrix}$$

where $x_j = (1-\delta_j)y$ and $\sigma_{xj}^2 \equiv (1-\delta_j)^2\sigma_y^2$.

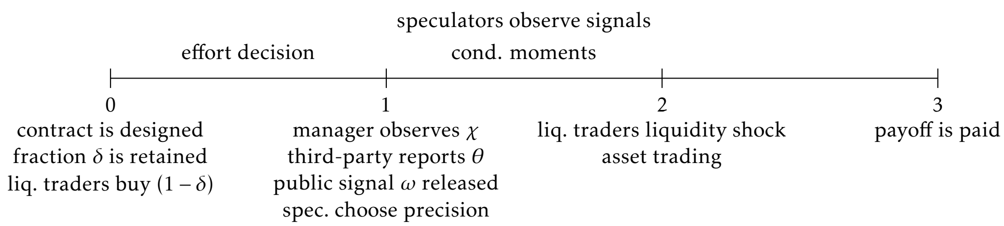
The interplay between public information and detection probability is characterized by letting the detection probability k_j be a function of $\tilde{\Sigma}_{wj}^{-1}$, such that $k_j(0) = 0$, $\lim_{\tilde{\Sigma}_{wj}^{-1} \to \infty} k\left(\tilde{\Sigma}_{wj}^{-1}\right) = \bar{k} \leq 1/2$, $k' > 0$, and $k'' < 0$. This reflects the possibility that the public information is released as the detection activity takes place, in accordance to the interpretation concerning the presence of a regulator.

Writing the public and private signals as $w = \pi + \xi_w$ and $\eta_i = \pi + \xi_{\eta i}$ with $\xi_w \sim \mathcal{N}\left(0, \tilde{\Sigma}_w\right)$ and $\xi_{\eta i} \sim \mathcal{N}\left(0, \tilde{\Sigma}_{\eta i}\right)$, I make the following assumption for the covariances:

[12]The assumption that the mean of liquidity shock is zero is immaterial for almost all results. However, in order to match the adverse capital market reactions that occur when earnings restatements are announced, this assumption must be modified. I address capital market reactions in Appendix G.

$$\mathbb{E}\left[\xi_\theta \xi_{\eta i}\right] = \mathbb{E}\left[\xi_\epsilon \xi_{\eta i}\right] = \mathbb{E}\left[\xi_w \xi_{\eta i}\right] = \mathbb{E}\left[x \xi_{\eta i}\right] = 0$$
$$\mathbb{E}\left[\xi_\theta x\right] = \mathbb{E}\left[\xi_\epsilon x\right] = \mathbb{E}\left[\xi_w x\right] = 0$$
$$\mathbb{E}\left[\xi_\theta \xi_w\right] = \mathbb{E}\left[\xi_\epsilon \xi_w\right] = 0$$
$$\mathbb{E}\left[\xi_\theta \xi_\epsilon\right] = 0$$

The following timeline summarizes the sequence of events in the model:



4. Equilibrium

I adopt a rational expectations equilibrium according to Definiton 1. I note that in this model there is no systematic mispricing because, in equilibrium, speculators perfectly discount for the bias in the financial statement.

Definition 1 (Equilibrium). *Let $\left\langle m_j, \theta_j \right\rangle_{j=1}^{n}$ be arbitrary but consistent*[13] *bias and financial statement. A rational expectations equilibrium consists of portfolio allocations $(u_i)_{i\in[0,1]}$; asset pricing schedules $\langle p_j \rangle_{j=1}^{n}$ and discounts $\left\langle s_j \right\rangle_{j=1}^{n}$ such that:*

(i) Given the asset pricing schedules, financial statements and discounts, u_i solves speculator's i problem for every $i \in [0,1]$;

(ii) Market clearing: $\int_0^1 u_i \,\mathrm{d}i = x$;

[13] that is, the financial statement is given by the true state plus a bias and some noise.

(iii) Rational expectations: $s_j = \left(1+\alpha_j\right) m_j, \forall j \in \{1,\ldots,n\}$.

At time $t = 2$, speculators and liquidity traders trade claims on the payoffs in a competitive market. The speculators constitute the informed party in the market and choose their portfolios based on their current information about the payoffs. This information comes from four sources, namely, the financial statement, the public and private signals and the information revealed through prices. I restrict attention to equilibria in the class of linear prices, i.e., pricing function linear in payoffs:

$$\tilde{p} = a(\theta) + B(\theta)\pi + C(\theta)x$$

where $\tilde{p} \equiv [\tilde{p}_1,\ldots,\tilde{p}_n]'$ with $\tilde{p}_j = h_{0j} + \left(1+h_{1j}\right) r p_j$ and x is the shock hitting liquidity traders. From this pricing function, speculators elicit information about payoffs upon observing price, because $E_p \equiv \mathbb{E}[\pi|p] = B^{-1}(\tilde{p}-a)$ and $V_p \equiv \mathbb{V}[\pi|p] = \left(B^{-1}C\right)' B^{-1}C\Sigma_x^2$. Thus, at time 2, an arbitrary speculator i combines the publicly available financial statement and a public signal, pricing and his private signal to form the *posterior*:

$$\hat{\mu}_i \sim \mathcal{N}\left(\pi, \hat{\Sigma}_i\right)$$

where:

$$\hat{\Sigma}_i^{-1} = \tilde{\Sigma}^{-1} + \tilde{\Sigma}_w^{-1} + \tilde{\Sigma}_{\eta i}^{-1} + V_p^{-1}$$

$$\hat{\mu}_i = \hat{\Sigma}_i\left(\tilde{\Sigma}^{-1}\tilde{\theta} + \tilde{\Sigma}_w^{-1} w + \tilde{\Sigma}_{\eta i}^{-1}\eta_i + V_p^{-1}E_p\right)$$

With this information set, speculator i solves:

$$\max_{u_i} \mathbb{E}_2\left[-\exp(-\rho W_i)|\eta_i,\tilde{p}\right]$$

s.t.

$$W_i = rW_{0i} + u_i'(\pi - \tilde{p}) + \text{transfers}$$

Existence and uniqueness in the linear class follow from Admati (1985) and, in the context of the model that I present, the pricing function is also linear in the financial statement[14]. The uniquely determined coefficients are given by:

[14] This result is shown in Appendix A.

$$
\begin{aligned}
&a = A_1\left(\theta - s + \bar{\epsilon}\right) + A_2 w, \quad A_1 \equiv \hat{\Sigma}_a \tilde{\Sigma}^{-1}, \quad A_2 \equiv \hat{\Sigma}_a \tilde{\Sigma}_w^{-1} \\
&B = \hat{\Sigma}_a \left(\Psi^{-1} + V_p^{-1}\right) \\
&C = -\rho \Psi \hat{\Sigma}_a \left(\Psi^{-1} + V_p^{-1}\right)
\end{aligned}
$$

where $\Psi^{-1} \equiv \int_0^1 \tilde{\Sigma}_{\eta i}^{-1}\, \mathrm{d}i$ is the average signal precision, $\hat{\Sigma}_a^{-1} \equiv \tilde{\Sigma}^{-1} + \Psi^{-1} + V_p^{-1} + \tilde{\Sigma}_w^{-1}$ is the average posterior precision, and $V_p = \left(B^{-1}C\right)' B^{-1} C \Sigma_x^2 = \rho^2 \Psi' \Psi \Sigma_x^2$.

I refer to the weight of the financial statement in speculators' average expectation, expressed by $\tilde{\Sigma}_j^{-1}/\hat{\Sigma}_{aj}^{-1}$, as *financial statement informativeness*. The pricing function reflects the fact that managers think themselves capable of increasing the price through manipulation of the financial statement. The perceived impact of manipulation on price is given by the financial statement informativeness; the higher the financial statement informativeness, the more speculators rely on the financial statement when forming their expectations, the greater the manager's perceived ability of affecting price through manipulation.

Information choice

Going backwards in time, each speculator anticipates his portfolio decision, as a function of the posterior available at such time, and decides on his capacity allocation for uncertainty reduction in order to maximize his utility. The optimal choice of payoff j signal precision by speculator i is given by:

$$
\tilde{\Sigma}_{\eta il}^{-1} = K + \frac{1}{n} \sum_j \left(\tilde{\Sigma}_j^{-1} + \tilde{\Sigma}_{wj}^{-1}\right) - \left(\tilde{\Sigma}_l^{-1} + V_{pl}^{-1} + \tilde{\Sigma}_{wl}^{-1}\right)
$$

The solution involves learning about every payoff[15] and reducing more of the uncertainty of payoffs about which he is more uncertain[16]. Moreover, every speculator makes the same information choice. As a result, individual posterior pre-

[15] As I show in Appendix E, a version of the entropy learning technology delivers specialized learning if paired with CARA utility. The aggregate learning result is similar nonetheless.

[16] I present the derivation of this result in Appendix B.

cisions are equal to the average precision. Letting $\hat{\Sigma}_{al}^{-1} \equiv \int_0^1 \hat{\Sigma}_{il}^{-1}\, \mathrm{d}i$ be the average posterior precision for payoff l, it follows that $\hat{\Sigma}_{al}^{-1} = K + \frac{1}{n}\sum_j \left(\tilde{\Sigma}_j^{-1} + \tilde{\Sigma}_{wj}^{-1}\right)$. Equivalently:

$$\tilde{\Sigma}_l^{-1} + \tilde{\Sigma}_{wl}^{-1} + \Psi_l^{-1} + \rho^{-2}\sigma_{xl}^{-2}\Psi_l^{-2} = K + \frac{1}{n}\sum_j \left(\tilde{\Sigma}_j^{-1} + \tilde{\Sigma}_{wj}^{-1}\right)$$

and this last equation determines $\Psi_l^{-1} \equiv \int_0^1 \tilde{\Sigma}_{\eta i}^{-1}\, \mathrm{d}i$ implicitly, being it decreasing in the amount of public information available.

Manager's manipulation choice

Following Holmström and Tirole (1993), I express the compensation as $T_j = q_{0j} + q_{1j} z_j$, where z_j is a normalized performance measure z_j defined as:

$$\begin{aligned} z_j &\equiv \frac{\tilde{p}_j + A_{1j}s_j - \left(A_{1j} + A_{2j} + B_j\right)\bar{\epsilon}_j}{\beta_j\left(A_{1j} + A_{2j} + B_j\right)} \\ &= e_j + \beta_j^{-1}\chi_j + \frac{A_{1j} - \left(A_{2j} + B_j\right)\alpha_j}{\beta_j\left(A_{1j} + A_{2j} + B_j\right)} m_j - \frac{A_{1j}}{\beta_j\left(A_{1j} + A_{2j} + B_j\right)}\xi_{\theta j} \\ &\quad + \frac{A_{2j}}{\beta_j\left(A_{1j} + A_{2j} + B_j\right)}\xi_{wj} + \frac{A_{2j} + B_j}{\beta_j\left(A_{1j} + A_{2j} + B_j\right)}\xi_{\epsilon j} + \frac{C_j}{\beta_j\left(A_{1j} + A_{2j} + B_j\right)} x \end{aligned}$$

The transformed variable z_j is constructed from public information and it does not depend on the terms of the contract. I refer to q_{1j} as the contract performance sensitivity[17]. I let $\phi_{state,j} \equiv \frac{\tilde{\Sigma}_j^{-1}}{\hat{\Sigma}_{aj}^{-1}}$, $\phi_{publ,j} \equiv \frac{\tilde{\Sigma}_{wj}^{-1}}{\hat{\Sigma}_{aj}^{-1}}$, $\phi_{priv,j} \equiv \frac{\Psi_j^{-1}}{\hat{\Sigma}_{aj}^{-1}}$ and $\phi_{feed,j} \equiv \frac{\rho^{-2}\sigma_{xj}^{-2}\Psi_j^{-2}}{\hat{\Sigma}_{aj}^{-1}}$ be the shares of information on the market that is conveyed through the finan-

[17] The original contract can be retrieved:

$$\tilde{h}_{1j} = \frac{q_{1j}}{\beta_j\left(A_{1j} + A_{2j} + B_j\right)}, \quad h_{0j} = \frac{1}{\beta_j\left(A_{1j} + A_{2j} + B_j\right) - q_{1j}}\left[\left(A_{1j} + A_{2j} + B_j\right)\left(\beta q_{0j} - \bar{\epsilon}_j q_{1j}\right) + q_{1j}A_{1j}s_j\right]$$

cial statement, through public information, through private information and the feedback through prices respectively and I define:

$$L_j = \beta_j^{-2}\Big[V_{\chi j} + \big(1 + \phi_{priv,j}\big)\hat{\Sigma}_{aj} + \Sigma_{\epsilon j} - 2\Sigma_{\epsilon j}\phi_{state,j} + \rho^2\sigma_x^2\hat{\Sigma}_{aj}^2\Big] \tag{1}$$

$$N_j = \beta_j^{-1}\big(1 + \alpha_j\big)\phi_{state,j} - \beta_j^{-1}\alpha_j \tag{2}$$

The moments of compensation T_j are $\mathbb{E}\big[T_j\big] = q_{0j} + q_{1j}\big(e_j + N_j m_j\big)$ and $\mathbb{V}\big[T_j\big] = q_{1j}^2 L_j$. Thus L_j is a measure of the uncertainty that the manager faces under the contract. As for N_j, it is the marginal impact of managerial manipulation on price perceived by the manager. Once weighed by the contract performance sensitivity, it is the perceived marginal benefit for the manager of biasing the financial statement[18]. Lemma 1 presents the manager's optimal choice of effort and report's bias for an arbitrary contract.

Lemma 1. *Let $\big(q_{0j}, q_{1j}\big)$ be an arbitrary contract. The optimal effort and report's bias choices are $e_j^* = \dfrac{q_{1j}}{c_{ej}}$ and $m_j^* = \max\Big\{0, \kappa_j^{-1}\big(q_{1j}N_j - k_j g_j\big)\Big\}$, where $\kappa_j \equiv \gamma k_j\big(1 - k_j\big)g_j^2$ is the incremental impact of manipulation on the risk faced by the manager.*

Since both effort and report's bias are increasing in the contract performance sensitivity, it will be used by the entrepreneur to balance those two competing forces on the payoff. Moreover, the incentive for the manager to manipulate brought about by the pay for performance is weighted by the marginal impact of manipulation perceived by the manager which, in turn, is increasing in the financial statement informativeness $\phi_{state,j}$. Moreover, for a fixed contract and sufficiently small financial statement informativeness, the report's bias would not respond to incremental changes in the contract performance sensitivity. I also observe that the uncertainty faced by the manager, L_j, only affects his choices through the contract performance sensitivity[19] q_{1j}.

Like Goldman and Slezak (2006), my model features "signal jamming" equilibria (e.g, Narayanan, 1985; Fudenberg and Tirole, 1986; Stein, 1989; Holmström,

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where $\tilde{h}_{1j} \equiv \dfrac{h_{1j}}{r\big(1 + h_{1j}\big)}$.

[18]For simplicity of language, I refer to N_j as the marginal benefit of manipulation perceived by the manager.

[19]I show later that indeed the contract depends on the uncertainty faced by the manager.

1999) in which the manager undertakes a costly action (manipulation) in order to bias upwards speculators' valuations and by doing so increase the price but, in equilibrium, valuations are not biased. For sufficiently high levels of performance pay, there cannot exist equilibrium with no manipulation in which speculators take the report at face value; if speculators take the report at face value, then the manager would rather manipulate because by doing so he would face higher stock prices, increasing thus his pay.

Contract

Anticipating how the manager chooses effort and report's bias as functions of the contract, the entrepreneur chooses the terms of the contract to maximize his revenue, that is, the sum of the amount he receives by selling a stake $1-\delta_j$ to liquidity traders plus the discounted value of the stake δ_j of the payoff that he holds. Lemma 2 presents the terms of the contract chosen by the entrepreneur.

Lemma 2. *Let* $\underline{q}_j \equiv \left(\gamma L_j + c_{ej}^{-1} + \kappa_j^{-1} N_j^2\right)^{-1} \left(\beta_j c_{ej}^{-1} - \alpha_j \kappa_j^{-1} N_j\right)$ *and* $\bar{q}_j \equiv \left(\gamma L_j + c_{ej}^{-1}\right)^{-1} \beta_j c_{ej}^{-1}$. *The entrepreneur's optimal choice of performance sensitivity is given by:*

$$q_{1j}^{*} = \begin{cases} 0 & \text{if} \quad k_j = 0,\ N_j > 0 \\ \bar{q}_j & \text{if} \quad k_j > 0,\ N_j > 0,\ \bar{q}_j N_j < k_j g_j \qquad \text{or} \quad N_j \leq 0 \\ N_j^{-1} k_j g_j & \text{if} \quad k_j > 0,\ N_j > 0,\ \underline{q}_j N_j < k_j g_j \leq \bar{q}_j N_j \\ \underline{q}_j & \text{if} \quad k_j > 0,\ N_j > 0,\ \underline{q}_j N_j \geq k_j g_j \end{cases} \tag{3}$$

The entrepreneur sets q_{0j} *in order to satisfy the manager's participation constraint.*

The solution to the entrepreneur's problem evidences how the contract performance depends on the marginal benefit of manipulation perceived by the manager. Low values of N_j allow the entrepreneur to implement the contract that he would set in the absence of manipulation. As N_j grows large, this contract ceases to be optimal, an the entrepreneur chooses the maximum contract performance sensitivity possible while still avoiding manipulation. For even larger values of N_j, the entrepreneur choose the contract that balances out effort and manipulation.

5. Comparative statics

In this section I present some comparative statics results that show how the report's bias and the contract performance sensitivity would vary in a cross-section of payoffs. I defer to the next section the results concerning policy related variables.

In Proposition 1, I restrict attention to a parametrization that ensures that manipulation occurs in equilibrium and derive comparative statics results for the contract performance sensitivity and report's bias with respect to manager's risk aversion γ, disutility of effort c_{ej}, productivity β_j and uncertainty V_{χ_j}.

Proposition 1. *Suppose that $k_j > 0$, $N_j > 0$ and $\underline{q}_j N_j > k_j g_j$. The following comparative statics results hold:*

(i) $\frac{\mathrm{d}q_{1j}^*}{\mathrm{d}\gamma}$ *can be either positive or negative.* $\frac{\mathrm{d}m_j^*}{\mathrm{d}\gamma}$ *can be either positive or negative, being negative if* $\frac{\mathrm{d}q_{1j}^*}{\mathrm{d}\gamma} < 0$.

(ii) $\frac{\mathrm{d}q_{1j}^*}{\mathrm{d}c_{ej}} < 0$ *and* $\frac{\mathrm{d}m_j^*}{\mathrm{d}c_{ej}} < 0$.

(iii) $\frac{\mathrm{d}q_{1j}^*/\beta_j}{\mathrm{d}\beta_j} > 0$ *and* $\frac{\mathrm{d}m_j^*}{\mathrm{d}\beta_j} > 0$.

(iv) $\frac{\mathrm{d}q_{1j}^*}{\mathrm{d}V_{\chi_j}} < 0$ *and* $\frac{\mathrm{d}m_j^*}{\mathrm{d}V_{\chi_j}} < 0$.

The intuition for (i) is that a higher risk aversion increases the cost of inducing effort and this force leads the entrepreneur to decrease the contract performance sensitivity. On the other hand, biasing the report would become costlier to the manager because of the risk of being caught (and fined) for manipulating; this would cause the entrepreneur to increase the contract performance sensitivity. This ambiguous result for the contract performance sensitivity translates into an ambiguous result for the report's bias as well, unless the contract performance sensitivity decreases with an increase in risk aversion; in such case the report's bias decreases. For (ii), for a given contract performance sensitivity, an increase in the cost of effort leads to a higher disutility for the manager; as it becomes costlier to induce effort, the entrepreneur responds by lowering the contract performance sensitivity which, in turn, induces the manager to decrease the report's bias in equilibrium.

Regarding (iii), for a given contract, an increase in the productivity raises the effort benefit and at the same time lowers the uncertainty the manager faces. Thus the entrepreneur responds by raising the contract performance sensitivity and as consequence the manager reports the statement with a larger bias. In (iv), for a given contract, I have an increase in the uncertainty faced by the manager and this causes the entrepreneur to reduce the contract performance sensitivity. This would result in smaller bias in the statement reported by the manager.

In Proposition 2, I present comparative statics results for variables related to the information structure, like the total amount of information $\hat{\Sigma}_{aj}^{-1}$. I also include the impact of changes in speculators' risk aversion ρ and liquidity on the contract performance sensitivity and report's bias because they enter into the information conveyed through prices; as they change, the information profile in the economy effectively changes as a result.

Proposition 2. *Suppose that $k_j > 0$, $N_j > 0$ and $\underline{q}_j N_j > k_j g_j$. The following comparative statics results hold:*

(i) For fixed shares of information conveyed by the financial statement and by public information, both contract performance sensitivity and report's bias are increasing in the amount of information. According to the utilitarian criterion, an increase in the amount of information is welfare improving.

(ii) Both contract performance sensitivity and report's bias are decreasing in liquidity.

(iii) Both contract performance sensitivity and report's bias are decreasing in the speculators' risk aversion ρ.

Regarding (i), I first note that while the shares of information conveyed by the financial statement and by public information are assumed to be constant, the share of private information is not and it does decrease as the total amount of information increases, because the ratio $\frac{\phi_{feed,j}}{\phi_{priv,j}}$ is increasing in the amount of private information. Thus an increase in the amount of information decreases the uncertainty faced by the manager under the contract; the insider responds by increasing the contract performance sensitivity which, in turn, leads to a larger report's bias.

An increase in the total amount of information is also welfare improving. On the one hand, it decreases the uncertainty faced by the manager under the contract and reduces the informational advantage of speculators, increasing thus entrepreneurs' value. On the other hand, the decrease in speculators' value is always more than compensated by the decrease in liquidity traders' loss. This constitutes one of the situations in which welfare improvements and larger report's bias occur concomitantly.

The logic behind (ii) and (iii) are similar. A higher liquidity makes the price less revealing, which induces the speculator to acquire more information of such asset. The uncertainty faced by the manager under the contract rises with the increase of private information. The entrepreneur's response to this increased cost of inducing effort is to reduce the contract performance sensitivity. Since the report's bias is only affected by liquidity through the contract, it decreases as well. Similarly, if speculators' become more risk averse, they will lower their position in the risky assets which, in turn, makes prices less revealing. This allows them to acquire more private information and the aforementioned result follows.

6. Main result: policy implications

This section contains the main result. I characterize the outcome of changes in variables related to policies attempting to curb manipulation. I present in Proposition 3 the outcome of changes in the incremental penalty g_j and the incremental resource cost α_j.

Proposition 3. *Suppose that $k_j > 0$, $N_j > 0$ and $\underline{q}_j N_j > k_j g_j$. The following comparative statics results hold:*

(i) $\dfrac{\mathrm{d}q^*_{1j}}{\mathrm{d}g_j} > 0$. *If* $\dfrac{\mathrm{d}q^*_{1j}}{\mathrm{d}g_j}\dfrac{g_j}{q^*_{1j}} < 1$, *then* $\dfrac{\mathrm{d}m^*_j}{\mathrm{d}g_j} < 0$; *if* $\dfrac{\mathrm{d}q^*_{1j}}{\mathrm{d}g_j}\dfrac{g_j}{q^*_{1j}} > 2$, *then* $\dfrac{\mathrm{d}m^*_j}{\mathrm{d}g_j} > 0$.

(ii) There exist ϕ'_j and ϕ''_j such that:

$\dfrac{\mathrm{d}q^*_{1j}}{\mathrm{d}\alpha_j} > 0$ *if* $\phi_{state,j} < \phi'_j$, *and* $\dfrac{\mathrm{d}q^*_{1j}}{\mathrm{d}\alpha_j} < 0$ *if* $\phi_{state,j} > \phi''_j$.

The report's bias is decreasing in the resource cost whenever the contract performance sensitivity is decreasing in this variable.

In (i), for a fixed contract, an increase in the incremental fine lowers the incentive the manager has for manipulating; the entrepreneur would raise the contract performance sensitivity. Concerning the manager, the competing forces coming from the higher incremental fine and from the higher contract performance sensitivity render an ambiguous result for the report's bias which can be settled in some special cases. For instance, if the elasticity of the contract performance sensitivity with respect to the incremental fine is sufficiently high, it overtakes the direct impact of the incremental fine on manipulation and the net effect is an increase in the report's bias. Thus the model suggests that a policy designed to curb manipulation by raising the incremental fine may fail to achieve its end.

The increase in the resource cost of manipulation considered in (ii) raises the cost of manipulation faced the entrepreneur but at the same time reduces the manager's perceived benefit of manipulation. Thus the response by the entrepreneur is generally ambiguous. However, the impact on manager's perceived benefit of manipulation is increasing in the financial statement informativeness. For sufficiently small values of financial statement informativeness the entrepreneur raises the contract performance sensitivity while for sufficiently high values the opposite holds true. For the manager, the direct impact on his perceived benefit and the indirect effect through the contract are reinforcing when the financial statement informativeness is sufficiently high.

I follow Goldman and Slezak (2006) and interpret the separation of the provision of certified accounting services from the provision of consulting services, as set in Title II of the Sarbanes-Oxley Act, as an instance of increase in the incremental resource cost of manipulation in the model. This constitutes the elimination of one of the ways by which manipulation is possible; hence, in order to manipulate, the manager would have to find other costlier alternatives. However, this measure worsens welfare in both models. Thus, my results suggest that the reduced share of incentive based compensation and earnings manipulation observed empirically after SOX bill was passed (Cohen et al., 2007, 2008) could be undesirable from a welfare perspective.

Increased scrutiny and public information

I start the analysis of scrutiny and the resulting provision of public information by considering its impact on the contract performance sensitivity and report's bias as shown in Proposition 4.

Proposition 4. *Suppose that* $0 < k_j < 1/3$*,* $N_j > 0$ *and* $\underline{q}_j N_j > k_j g_j$*. Let the total amount of information and the share conveyed by financial statements be fixed. The contract performance sensitivity is increasing in scrutiny. If manager's risk aversion is sufficiently low, the report's bias is increasing in scrutiny; if manager's risk aversion is sufficiently high and* $\phi_{publ,j}$ *sufficiently low, the report's bias is decreasing in scrutiny.*

In the scenario described in Proposition 4, public information crowds-out private information. This is consistent with the fact that if a regulator scrutinizes and reveals public information, observing it would require capacity from speculators. For the insiders, public information disclosed as the company is scrutinized would decrease the uncertainty faced by the manager under the contract. This, along with the higher detection probability would prompt insiders to raise the contract performance sensitivity.

Managers, however, may either increase or decrease the report's bias depending on the prevailing effect, the higher contract performance sensitivity or the increase in detection probability. Conditions guaranteeing that the report's bias decreases or increases depend on the contract elasticity with respect to changes in the incremental impact of manipulation on the risk faced by the manager, which is proportional to manager's risk aversion. If manager's risk aversion is sufficiently low, the aforementioned elasticity is high so that the increase in the contract performance sensitivity is sufficiently high to prevail over the higher detection probability and thus the report's bias increases. The report's bias decreases if the elasticity is sufficiently small (high values of manager's risk aversion) and the detection is very responsive so that the impact on the risk he faces under the contract is small in comparison. The latter happens for low values of $\phi_{publ,j}$.

I have regarded scrutiny and the release of public information as being an activity performed by a regulator. However, I still have not addressed the reason why managers would not gather and release information to market participants[20]. Even more fundamentally, I have not shown that it is desirable for a regulator to pursue this activity. Lemma 3 and Proposition 5 provide answers to both questions.

Lemma 3. *Suppose that the manager has limited resources to spend in improving the informativeness of the financial statement or gathering payoff information meant for public release. Assume further that the manager cannot decide whether to release the information after observing the realization; rather, once the manager decides to gather public information, the realization is observed to all. Then the manager releases no public information at all.*

The manager would never gather information for public release in this case, because it would only decrease the perceived marginal benefit of manipulation while increasing the uncertainty that he would face under the contract, as speculators would rely less on the financial statement (information which the manager has control).

[20]Obviously, this information, albeit public, would not be linked to scrutiny.

Proposition 5. *Let the total amount of information on the market and the financial statement informativeness be fixed. Scrutiny and the resulting public information is beneficial to insiders and harmful to speculators. According to a utilitarian view of welfare, it is welfare improving.*

The result concerning speculators is not surprising: they only benefit from a higher share of private information. As for insiders, they are affected by public information in three ways:

(i) liquidity traders loss: by displacing private information, liquidity traders loss decreases along with the decrease in the speculators' informational advantage. Since this loss is borne by insiders, a higher share of public information would make insiders better off through this channel.

(ii) uncertainty faced by the manager: more public information diminishes the uncertainty faced by the manager under the contract, reducing insiders' cost of inducing effort.

(iii) manipulation: for a fixed contract, a higher share of public information increases both the probability of manipulation detection and the incremental impact of manipulation on the risk faced by the manager. As a result, the report's bias decreases. Moreover, the impact on the report's bias prevails over the impact on the detection probability and on the risk; thus the cost of manipulation for the manager is decreasing in the share of public information.

It is clear then that insiders benefit from a higher share of public information. Furthermore, the beneficial effect for insiders dominates and public information is welfare improving.

Thus a regulator could improve welfare by scrutinizing and releasing public information. In my setup, a welfare maximizing regulator would do this as much as possible. However, as I have shown, speculators are harmed by the release of public information and, if possible they would try to limit the amount of scrutiny promoted by the regulator. More interestingly, even if limiting scrutiny proves impracticable, I shall show that they would benefit from influencing which managers should be scrutinized. While I do not model this underlying game, I study the outcome when this does happen in an extension of the model which is presented in the next section.

Financial statement informativeness

The enhancement of financial disclosure is another regulatory aspect of SOX. Indeed, Title IV prescribes internal controls to improve the accuracy of financial reports and it can be mapped into the financial statement informativeness in the model. Proposition 6 presents the comparative statics results obtained with respect to that variable. I shall view all quantities as functions of $\phi_{state,j}$, restricting attention to intervals in which manipulation occurs ($\underline{q}_j N_j \geq k_j g_j$).

Proposition 6. *Suppose that $k_j > 0$, $N_j > 0$ and $\underline{q}_j N_j \geq k_j g_j$. Let:*

$$\underline{c}_j \equiv \max\left\{ \beta_j^{-2}\kappa_j^{-1}\alpha_j\Big[1-(1+\alpha_j)\phi_{publ,j}\Big] \quad , \right.$$
$$\left. 2\beta_j^{-2}\kappa_j^{-1}k_j g_j(\alpha_j + k_j g_j)\left[1+\left(1+\frac{\beta_j^2\gamma\kappa_j L_j(1-\phi_{publ,j})}{k_j g_j(1+\alpha_j)}\right)^{1/2}\right]\right\}$$
$$\bar{c}_j \equiv \min\left\{ \frac{2\gamma(\alpha_j + 2k_j g_j)(1+\alpha_j)}{\gamma\kappa_j\Big(\hat{\Sigma}_{aj}+2\Sigma_{\epsilon j}\Big) - 2(\alpha_j+2k_j g_j)(1+\alpha_j)L_j(1-\phi_{publ,j})} \quad , \right.$$
$$\left. \frac{\gamma^2\kappa_j k_j g_j(\alpha_j+k_j g_j)\Big(\hat{\Sigma}_{aj}+2\Sigma_{\epsilon j}\Big)^2 + \beta_j^2\gamma(\alpha_j+2k_j g_j)^2(1+\alpha_j)^2 L_j(1-\phi_{publ,j})}{\beta_j^2(\alpha_j+2k_j g_j)(1+\alpha_j)\Big[\kappa_j\Big(\hat{\Sigma}_{aj}+2\Sigma_{\epsilon j}\Big)-(\alpha_j+2k_j g_j)(1+\alpha_j)\Big]}\right\}$$

Assume that $\underline{c}_j \leq c_{ej}^{-1} < \bar{c}_j$. The contract performance sensitivity is decreasing in the financial statement informativeness. If $\gamma \leq \dfrac{(1+\alpha_j)\rho\sigma_{xj}}{k_j^{1/2}(1-k_j)^{1/2}g_j}$, then manipulation is concave in the financial statement informativeness.

This result suggests that the contract performance sensitivity and manipulation are not necessarily positively correlated. That is, among the managers who manipulate, while manipulation and misreporting are positively correlated for highly informative financial statements, they are negatively correlated for little informative statements. The reason for that is as follows. As the financial statement becomes more informative, the uncertainty faced by the manager under the contract decreases while the his perceived benefit increases. Provided that the cost of effort is not too low or too high, the latter consideration prevails to the entrepreneur, who responds by lowering the contract performance sensitivity and does so at higher rates if the manager is not too risk averse.

For little informative statements, the effect coming from the higher perceived benefit prevails and the amount of manipulation is increasing in financial statement informativeness; for highly informative financial statement values, the impact coming from the contract eventually prevails and, as a result, manipulation is decreasing in financial statement informativeness. This is yet another possible explanation for the decrease in both reports' bias and performance pay that followed the SOX bill. Although this result can be interpreted as the outcome of the aforementioned policy, it cannot be thought as being a source of cross sectional variation. In this exercise, I assumed that the detection probability is constant and, in the cross section, it translates into uniform probability of detection, which may not correspond to the scrutiny pattern pursued by the regulator. This motivates the extension of the model, in which scrutiny is endogenously determined and, in that context, address cross sectional differences in financial statement informativeness.

7. Extension: endogenous scrutiny

In this section I provide an extension to the model by allowing for endogenous scrutiny. As I have shown, speculators are harmed by the release of public information and they may attempt to mitigate their surplus losses by influencing the scrutiny activity. This section shows that they do have a preference about the way in which scrutiny is distributed across the different payoffs.

Because speculators are symmetric, their indirect utility does not depend on their identity. Moreover, the indirect utility is decreasing on $\tilde{\Sigma}_{wj}^{-1}$. The reason is that as the report's bias is correctly anticipated (and priced) in equilibrium, the speculator's perspective is a purely informational one. As more public information becomes available, each speculator relies less on his private information, which is the source for his utility gains from trading with heterogeneously informed and uninformed parties. This information, which is public, decreases the differentiation brought about by private information, reducing his informational advantage and his utility as a result. This produces a tension between speculators interests who benefit from no detection at all, and insiders who benefit from detection, as the terminal value is increasing in the detection probability. In this section, I assume that a minimum level of scrutiny (capacity K_2) is required while speculators are able to decide how it is allocated among the payoffs. The regulator's problem is formulated then as the maximization of the indirect utility subjected to the constraint $\sum_j \tilde{\Sigma}_{wj}^{-1} = nK_2$. Its solution is characterized by[21]:

$$\tilde{\Sigma}_{wj}^{-1} = K_2 + \frac{1}{n}\sum_i \tilde{\Sigma}_i^{-1} - \tilde{\Sigma}_j^{-1}$$

The solution of the regulator's problem smooths out any difference in the precision of the information publicly available of the payoffs. Managers whose financial statements are less precise would suffer more scrutiny and face a higher detection probability. Essentially, this policy prescribes that companies with higher information asymmetry between managers and market participants should be scrutinized at a higher intensity. Given the result of the speculator's capacity allocation problem, the average amount of private information is constant across payoffs, i.e., $\Psi_j^{-1} + \rho^{-2}\sigma_x^{-2}\Psi_j^{-2} = K$.

Would the contract differ in case the only difference between the payoffs were the noise in the financial statement? The answer is affirmative. While it is true that the regulator's policy equalizes the sum of public information and information conveyed by the financial statement for all payoffs, from a contracting perspective those two sources of information are substantially different: the prior (financial statement) is affected by manipulation while the signal is not.

I shall view all quantities as functions of $\phi_{state,j}$ and I let $[\phi_{1j}, \phi_{2j}]$ denote an interval over which $q_{1j}^* N_j \geq k_j g_j$ holds (with equality at the endpoints) and consider variations $\mathrm{d}\tilde{\Sigma}_i^{-1}$ such that $\sum_i \mathrm{d}\tilde{\Sigma}_i^{-1} = 0$ since they preserve the total amount of information in the market while varying the information conveyed by the financial statement of the assets. Proposition 7 characterizes the contract performance sensitivity and the bias (functions q_{1j}^* and m_j^*) on this interval.

Proposition 7. *Let* $\bar{\gamma}_j \equiv \left(\frac{\beta_j^2}{2\Sigma_{\epsilon j} g^2}\right)^{1/2}\left[\frac{\alpha_j N_j(\phi_{1j})}{g k_j(\phi_{1j})}\frac{\mathrm{d}k_j^{-1}(1-k_j)^{-1}N_j}{\mathrm{d}\phi_{state,j}}(\phi_{1j}) + \frac{\mathrm{d}k_j^{-1}(1-k_j)^{-1}N_j^2}{\mathrm{d}\phi_{state,j}}(\phi_{1j})\right]^{1/2}$. *If* $\gamma \leq \bar{\gamma}_j$, *then* q_{1j}^* *is decreasing on the interval* $[\phi_{1j}, \phi_{2j}]$; *otherwise, there exists* $\phi_{qj} \in (\phi_{1j}, \phi_{2j})$, *such that* q_{1j}^* *is increasing on* $[\phi_{1j}, \phi_{qj}]$ *and decreasing on* $[\phi_{qj}, \phi_{2j}]$.

There exists $\phi_{mj,1} \in (\phi_{1j}, \phi_{2j})$ *such that* m_j^* *is increasing on* $[\phi_{1j}, \phi_{mj,1}]$. *Also,* $\phi_{mj,1} > \phi_{qj}$ *if* ϕ_{qj} *exists.*

Letting $\Phi_{mj} = \{\phi_{mj,1}, \phi_{mj,2}, \ldots\}$, *with* $\phi_{mj,1} < \phi_{mj,2} < \cdots$ *be the set of local extrema of* m_j^*, *it follows that* $\phi_{mj,k}$ *is a local maximum if* k *is odd and a local minimum if* k *is even. Thus, letting* $\phi_{mj,0} \equiv \phi_{1j}$, m_j^* *is increasing on any interval* $(\phi_{mj,2k}, \phi_{mj,2k+1})$ *with*

[21] The derivation of this solution is presented in Appendix C.

$k = 0, 1, 2, \ldots$ and decreasing on any interval $(\phi_{mj,2k+1}, \phi_{mj,2k+2})$ with $k = 0, 1, 2, \ldots$.

Corollary 1. *Let $\phi_{mj,0} \equiv \phi_{qj}$ if ϕ_{qj} exists and $\phi_{mj,0} \equiv \phi_{1j}$ otherwise. On any interval $(\phi_{mj,2k}, \phi_{mj,2k+1})$ with $k = 0, 1, 2, \ldots$, the report's bias m_j^* and the contract performance sensitivity q_j^* move in opposite directions, that is, m_j^* increases while q_j^* decreases.*

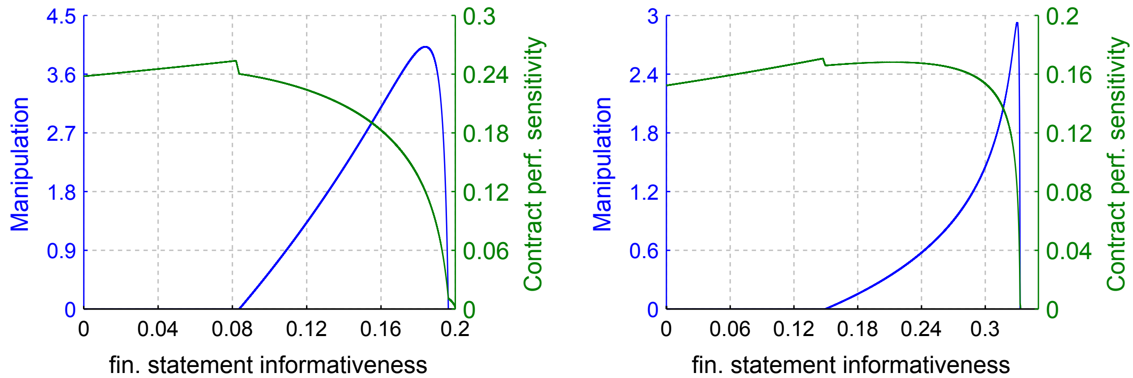
Proposition 7 and Corollary 1 show that measures that exogenously cap the contract performance sensitivity while ignoring the heterogeneity in the informativeness of financial statements will not generally be addressing managers reporting the most biased statements. Also of note is the fact that managers of payoffs whose financial statements are the least informative are not the ones reporting the most biased statements either. Managers who report with the largest bias are those whose financial statements informativeness are "average".

Corollary 2. *On any interval in which the report's bias increases while the contract performance sensitivity decreases, detection probability is decreasing.*

The optimal scrutiny decision by a regulator acting according to speculators' interests implies that the detection probability is decreasing in the financial statement informativeness. As a result, Corollary 2 indicates that managers whose financial statements are the least informative, besides being some of the managers who report with smallest bias, are those whose manipulation is most detected, while managers whose reports contain the largest bias are subject to intermediate detection probability only.

Figure 1 depicts two examples[22] which, depending on the set of parameter restrictions, entail either a decreasing contract performance sensitivity on the whole set in which the bias is positive or an increasing and then decreasing contract performance sensitivity on such set. Regardless of the case, companies with the most informative financial statements would have lower contract performance sensitivities in equilibrium. If larger companies are also characterized by highly informative financial statements, this result would be in accordance to the empirical evidence that larger companies have lower contract performance sensitivities (Murphy, 1999).

[22] In both cases, $k(\tilde{\Sigma}_{wj}^{-1}) = \frac{1}{4}\frac{\exp\left(\tilde{\Sigma}_{wj}^{-1/2}\right) - 1}{\exp\left(\tilde{\Sigma}_{wj}^{-1/2}\right) + 1}$, $\beta = 1$, $c_e = .2$, $\alpha = .05$, $V_{\chi j} = .5$, $\rho = .5$, $\Sigma_j = 2$, $\Sigma_{\epsilon,j} = .8$, $\frac{1}{n}\sum \tilde{\Sigma}_j^{-1} + K_2 = 1$, $K = 4$.
Case $F(\phi_{1j}) < 0$: $g = .1$, $\gamma = 10$.
Case $F(\phi_{1j}) > 0$: $g = .2$, $\gamma = 15$.



(a) $F(\phi_{1j}) < 0$

(b) $F(\phi_{1j}) > 0$

Figure 1: Bias and contract performance sensitivity

Regarding the first case (Figure 1a), I consider four regions for the financial statement informativeness:

(i) little informative financial statement $\phi_{state} \in (0, 0.08)$: this region is characterized by high detection probability, even though the manager does not engage in manipulation; the contract performance sensitivity is the highest.

(ii) moderately informative financial statement $\phi_{state} \in (0.08, 0.13)$: this region is characterized by average detection probability, high but decreasing contract performance sensitivity and low but increasing bias.

(iii) highly informative financial statement $\phi_{state} \in (0.13, 0.18)$: this region is characterized by low detection probability, average and decreasing contract performance sensitivity, and high and increasing bias.

(iv) most informative financial statement $\phi_{state} \in (0.18, 0.19)$: this region is characterized by the lowest detection probability, low and decreasing contract performance sensitivity, and a highly variable bias (decreasing from its highest level to zero).

This example illustrates a case in which the highest contract performance sensitivity is paid to managers who do not manipulate at all. Thus, depending on the threshold, an attempt to cap the contract performance sensitivity may not change the bias in the financial statements. Moreover, even if a different cap is imposed on different intervals of contract performance sensitivity, it would result in curbing managers who bias just a little already, as it can be seen from the fact that bias and contract performance sensitivity move on opposite directions on the set of moderately and highly informative financial statements.

Regardless of the case, the report's bias is zero for financial statements low in information content and at such region scrutiny is at its highest level.

Conclusion

In this paper, I have shown that the information profile in the economy affects the managerial manipulation choice through both the benefit and risk channels. As a result, insiders would take the information profile into account in setting the contract. The very information profile is endogenous, depending on speculators' information choice, which in turn depends only on speculators' risk aversion, liquidity and the amount information from other sources, namely financial statement and public information. This latter kind of information cannot be provided but by a regulator.

As speculators are harmed by public information, they may attempt to either limit scrutiny or to influence the regulator's choice of payoffs to scrutinize. This seems in consonance with the fact that speculators call for scrutiny at some companies. If they are successful, the resulting information profile is one in which companies with the least informative financial statements and highest performance pay levels will be those scrutinized the most.

The fact that bias may increase with scrutiny and public information suggests that caution is needed when facing a period of large bias being detected. In fact, this phenomenon may occur precisely because more scrutiny was exercised by a regulator. Finally, because the report's bias and the contract performance sensitivity may move in opposite directions, measures that establish an upper bound for the contract performance sensitivity may not target managers who report the most biased statements. The main message: the information profile cannot be ignored when designing policies in this regard.

Appendix A. Existence and uniqueness of a linear equilibrium

Lemma 4. *Fix the managers contracts* (h_0, H_1). *Let* $\tilde{p} = a + B\pi + Cx$. *The portfolio choice* $u_i^* = \rho^{-1}\hat{\Sigma}_i^{-1}(\hat{\mu}_i - \tilde{p})$ *solves speculator i's portfolio problem.*

Proof. Observe that $\tilde{p}$ was constructed from public information. According to speculator i's information set, his wealth is normally distributed with moments given by $\mathbb{E}[W_i|\eta_i,\tilde{p}] = rW_{0i} + u_i'(\hat{\mu}_i - \tilde{p}) + \mathbb{E}[\text{transfers}]$ and $\mathbb{V}[W_i|\eta_i,\tilde{p}] = u_i'\hat{\Sigma}_i u_i + \mathbb{V}[\text{transfers}]$. Using these facts, rewrite the problem:

$$\max_{u_i} -\exp\Big(-\rho\Big(rW_{0i} + u_i'(\hat{\mu}_i - \tilde{p}) + \mathbb{E}[\text{transfers}]\Big) \\ - \frac{\rho^2}{2}\Big(u_i'\hat{\Sigma}_i u_i + \mathbb{V}[\text{transfers}]\Big)\Big)$$

And $u_i^* = \rho^{-1}\hat{\Sigma}_i^{-1}(\hat{\mu}_i - \tilde{p})$ solves this maximization problem. □

Proposition 8. *Let θ be the managers' equilibrium actions*[23]*. There exists a unique linear equilibrium in the assets market. Moreover, the pricing function is linear in the financial statement.*

Proof. In solving the speculator's problem in Lemma 4, I postulated a linear pricing function. Here I show that within the class of linear pricing functions there exists a unique equilibrium and that such function is also linear in the financial statement.

Integrating the portfolio choice u_i^* over i and imposing market clearing, the pricing coefficients are uniquely determined:

$$a = \left(\tilde{\Sigma}^{-1} + \Psi^{-1} + V_p^{-1} + \tilde{\Sigma}_w^{-1}\right)^{-1}\left(\tilde{\Sigma}^{-1}\tilde{\theta} + \tilde{\Sigma}_w^{-1}w\right)$$
$$B = \left(\tilde{\Sigma}^{-1} + \Psi^{-1} + V_p^{-1} + \tilde{\Sigma}_w^{-1}\right)^{-1}\left(\Psi^{-1} + V_p^{-1}\right)$$
$$C = -\rho\Psi\left(\tilde{\Sigma}^{-1} + \Psi^{-1} + V_p^{-1} + \tilde{\Sigma}_w^{-1}\right)^{-1}\left(\Psi^{-1} + V_p^{-1}\right)$$

where $\Psi^{-1} \equiv \int_0^1 \tilde{\Sigma}_{\eta i}^{-1}\,\mathrm{d}i$. Since $V_p = \sigma_x^2\left(B^{-1}C\right)'B^{-1}C = \rho^2\sigma_x^2\Psi'\Psi$, the uniquely determined coefficients are expressed by:

[23]With an abuse of notation.

$$
\begin{aligned}
a &= A_1\tilde{\theta} + A_2 w \\
B &= \left(\tilde{\Sigma}^{-1} + \tilde{\Sigma}_w^{-1} + \Psi^{-1} + \rho^{-2}\sigma_x^{-2}\Psi'^{-1}\Psi^{-1}\right)^{-1}\left(\Psi^{-1} + \rho^{-2}\sigma_x^{-2}\Psi'^{-1}\Psi^{-1}\right) \\
C &= -\rho\Psi\left(\tilde{\Sigma}^{-1} + \tilde{\Sigma}_w^{-1} + \Psi^{-1} + \rho^{-2}\sigma_x^{-2}\Psi'^{-1}\Psi^{-1}\right)^{-1}\left(\Psi^{-1} + \rho^{-2}\sigma_x^{-2}\Psi'^{-1}\Psi^{-1}\right)
\end{aligned}
$$

where:

$$
\begin{aligned}
A_1 &= \left(\tilde{\Sigma}^{-1} + \tilde{\Sigma}_w^{-1} + \Psi^{-1} + \rho^{-2}\sigma_x^{-2}\Psi'^{-1}\Psi^{-1}\right)^{-1}\tilde{\Sigma}^{-1} \\
A_2 &= \left(\tilde{\Sigma}^{-1} + \tilde{\Sigma}_w^{-1} + \Psi^{-1} + \rho^{-2}\sigma_x^{-2}\Psi'^{-1}\Psi^{-1}\right)^{-1}\tilde{\Sigma}_w^{-1}
\end{aligned}
$$

□

Appendix B. Speculators' learning choice

Lemma 5. *As of time $t = 1$, speculator's i problem can be stated as* $\max\sum_j \ln\hat{\Sigma}_{ij}^{-1}$, *subject to* (NFC) *and* (AC).

Proof. Substituting for the optimal portfolio choice, speculator's i utility as of time $t = 1$ is given by:

$$
\begin{aligned}
&\mathbb{E}_1\mathbb{E}_2\left[-\exp\left(-\rho W_i\left(u_i^*\right)\right)|\eta_i,\tilde{p}\right] = \\
&\qquad\qquad \mathbb{E}_1\mathbb{E}_2\left[-\exp\left(-\rho\left(rW_{0i} + u_i^{*\prime}(\pi-\tilde{p}) + \text{transfers}\right)\right)|\eta_i,\tilde{p}\right] \\
&= -\exp\left(-\rho rW_{0i}\right)\mathbb{E}_1\mathbb{E}_2\left[\exp\left(-(\hat{\mu}_i-\tilde{p})\hat{\Sigma}_i^{-1}(\pi-\tilde{p}) + \rho\times\text{transfers}\right)|\eta_i,\tilde{p}\right] \\
&= -\exp\left(-\rho rW_{0i}\right)\mathbb{E}_1\left[\exp\left(-(\hat{\mu}_i-\tilde{p})\hat{\Sigma}_i^{-1}(\hat{\mu}_i-\tilde{p}) + \frac{1}{2}(\hat{\mu}_i-\tilde{p})\hat{\Sigma}_i^{-1}(\hat{\mu}_i-\tilde{p})\right.\right. \\
&\qquad\qquad \left.\left. -\rho\mathbb{E}[\text{transfers}] + \frac{\rho^2}{2}\mathbb{V}[\text{transfers}]\right)\right] \\
&= -\exp\left(-\rho rW_{0i} - \rho\mathbb{E}[\text{transfers}] + \frac{\rho^2}{2}\mathbb{V}[\text{transfers}]\right)\times \\
&\qquad \mathbb{E}_1\left[\exp\left(-\frac{1}{2}(\hat{\mu}_i-\tilde{p})'\hat{\Sigma}_i^{-1}(\hat{\mu}_i-\tilde{p})\right)\right]
\end{aligned}
$$

where the third equality follows from the fact that, according to time 2 speculator i's information set, π is the only random variable in the formula, with

$\pi \sim \mathcal{N}\left(\hat{\mu}_i, \hat{\Sigma}_i\right)$. As of time 1, according to speculator's i information set, $\hat{\mu}_i - \tilde{p}$ is a normally distributed random variable with the following moments:

$$\mathbb{E}[\hat{\mu}_i - \tilde{p}] = (I-B)\tilde{\theta} - a = 0$$
$$\mathbb{V}[\hat{\mu}_i - \tilde{p}] = (I-B)\tilde{\Sigma}(I-B)' - \hat{\Sigma}_i + \sigma_x^2 CC' - (I-B)\hat{\Sigma}_a \Sigma_w^{-1} \tilde{\Sigma}$$

Let $y \equiv \hat{\mu}_i - \tilde{p}$. For $y \sim \mathcal{N}(0, V)$:

$$\mathbb{E}[\exp(y'Fy + G'y + H)] = |I - 2VF|^{-1/2} \exp\left(\frac{1}{2}G'(I-2VF)^{-1} VG + H\right)$$

Here $F = -\frac{1}{2}\hat{\Sigma}^{-1}$, $G = 0$, $H = 0$. As of time $t = 1$, speculator's i utility function can be written as:

$$-\exp\left(-\rho r W_{0i} - \rho \mathbb{E}[\text{transfers}] + \frac{\rho^2}{2}\mathbb{V}[\text{transfers}]\right) \times$$
$$\det\left(\left[(I-B)\tilde{\Sigma}(I-B)' + \sigma_x^2 CC' - (I-B)\hat{\Sigma}_a \tilde{\Sigma}_w^{-1} \tilde{\Sigma}\right]\hat{\Sigma}_i^{-1}\right)^{-1/2}$$

Disregarding the exponential terms, which do not depend on the speculator's choice, and applying the transformation $-2\ln(-(\cdot))$, the objective function of the maximization becomes:

$$\sum_j \ln \hat{\Sigma}_{ij}^{-1} + \sum_j \ln\left[\left(1 - B_j\right)^2 \tilde{\Sigma}_j + \sigma_x^2 C_j^2 - \left(I - B_j\right)\hat{\Sigma}_{aj} \tilde{\Sigma}_{wj}^{-1} \tilde{\Sigma}_j\right]$$

Thus, it all boils down to maximizing $\sum_j \ln \hat{\Sigma}_{ij}^{-1}$. □

Proposition 9. *Each speculator chooses to learn about every payoff, learning more about payoffs which he initially knows less. Moreover, every speculator makes the same information choice. For each payoff l, speculator i chooses:*

$$\tilde{\Sigma}_{\eta il}^{-1} = K + \frac{1}{n}\sum_j \left(\tilde{\Sigma}_j^{-1} + \tilde{\Sigma}_{wj}^{-1}\right) - \left(\tilde{\Sigma}_l^{-1} + V_{pl}^{-1} + \tilde{\Sigma}_{wl}^{-1}\right)$$

Proof. Forming the Lagrangian:

$$\begin{aligned}\mathcal{L} = &\sum_j \ln\left(\tilde{\Sigma}_{\eta ij}^{-1} + \tilde{\Sigma}_j^{-1} + V_{pj}^{-1} + \tilde{\Sigma}_{wj}^{-1}\right) + \sum_j \lambda_j \tilde{\Sigma}_{\eta ij}^{-1} \\ &+ \zeta\left[nK - \sum_j \left(\hat{\Sigma}_{ij}^{-1} - \tilde{\Sigma}_j^{-1} - \tilde{\Sigma}_{wj}^{-1}\right)\right]\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial \tilde{\Sigma}_{\eta ij}^{-1}} = \left(\tilde{\Sigma}_{\eta ij}^{-1} + \tilde{\Sigma}_j^{-1} + V_{pj}^{-1} + \tilde{\Sigma}_{wj}^{-1}\right)^{-1} + \lambda_j - \zeta$$

$$\begin{aligned}\frac{\partial^2 \mathcal{L}}{\partial \tilde{\Sigma}_{\eta ij}^{-2}} &= -\left(\tilde{\Sigma}_{\eta ij}^{-1} + \tilde{\Sigma}_j^{-1} + V_{pj}^{-1} + \tilde{\Sigma}_{wj}^{-1}\right)^{-2} < 0 \\ \frac{\partial^2 \mathcal{L}}{\partial \tilde{\Sigma}_{\eta ij}^{-1} \tilde{\Sigma}_{\eta il}^{-1}} &= 0 \qquad \text{for } j \neq l\end{aligned}$$

Thus the Lagrangian is concave and so the solution is interior:

$$\tilde{\Sigma}_{\eta il}^{-1} = K + \frac{1}{n}\sum_j \left(\tilde{\Sigma}_j^{-1} + \tilde{\Sigma}_{wj}^{-1}\right) - \left(\tilde{\Sigma}_l^{-1} + V_{pl}^{-1} + \tilde{\Sigma}_{wl}^{-1}\right)$$

The optimal signal precision choice for asset l is negatively related to $\tilde{\Sigma}_l^{-1} + V_{pl}^{-1} + \tilde{\Sigma}_{wl}^{-1}$, so the speculator chooses to reduce more of the uncertainty of payoffs about which he is more uncertain. Moreover, the information choice does not depend on the identity of the speculator, it follows that they all make the same information choice. □

Appendix C. Solution to regulator's problem

The regulator's problem can be equivalently written as:

$$\max \sum_j \ln\left(\tilde{K} + \frac{1}{n}\sum_l \tilde{\Sigma}_l^{-1} + \Psi_j^{-1} + \rho^2\sigma_x^2\right)$$

s.t.

$$\sum_j \tilde{\Sigma}_{wj}^{-1} = nK_2$$

where $\tilde{K} \equiv K + K_2$.

Forming the Lagrangian:

$$\mathcal{L} = \sum_j \ln\left(\tilde{K} + \frac{1}{n}\sum_l \tilde{\Sigma}_l^{-1} + \Psi_j^{-1} + \rho^2\sigma_x^2\right) + \lambda\left(nK_2 - \sum_j \tilde{\Sigma}_{wj}^{-1}\right)$$

$$\frac{\partial\mathcal{L}}{\partial\tilde{\Sigma}_{wi}^{-1}} = \sum_j\left(\tilde{K} + \frac{1}{n}\sum_l \tilde{\Sigma}_l^{-1} + \Psi_j^{-1} + \rho^2\sigma_x^2\right)^{-1}\frac{\partial\Psi_j^{-1}}{\partial\tilde{\Sigma}_{wi}^{-1}} - \lambda$$

Recall that $\hat{\Sigma}_{al}^{-1} = K + \frac{1}{n}\sum_j\left(\tilde{\Sigma}_j^{-1} + \tilde{\Sigma}_{wj}^{-1}\right) = \tilde{K} + \frac{1}{n}\sum_j \tilde{\Sigma}_j^{-1}$ for every payoff l, which implies $\frac{\partial\hat{\Sigma}_{al}^{-1}}{\partial\tilde{\Sigma}_{wj}^{-1}} = \frac{\partial}{\partial\tilde{\Sigma}_{wj}^{-1}}\left(\tilde{\Sigma}_l^{-1} + \tilde{\Sigma}_{wl}^{-1} + \Psi_l^{-1} + \rho^2\sigma_x^2\Psi_l^{-2}\right) = 0$ for every payoff l and j. Thus $\frac{\partial\Psi_j^{-1}}{\partial\tilde{\Sigma}_{wj}^{-1}} = \frac{-1}{1+2\rho^{-2}\sigma_x^{-2}\Psi_j^{-1}}$ and $\frac{\partial\Psi_l^{-1}}{\partial\tilde{\Sigma}_{wj}^{-1}} = 0$ for $l \neq j$. Using these results:

$$\frac{\partial\mathcal{L}}{\partial\tilde{\Sigma}_{wi}^{-1}} = -\left(\tilde{K} + \frac{1}{n}\sum_l \tilde{\Sigma}_l^{-1} + \Psi_i^{-1} + \rho^2\sigma_x^2\right)^{-1}\frac{1}{1+2\rho^{-2}\sigma_x^{-2}\Psi_i^{-1}} - \lambda$$

$$\frac{\partial^2 \mathcal{L}}{\partial \tilde{\Sigma}_{wi}^{-2}} = -\left(\tilde{K} + \frac{1}{n}\sum_l \tilde{\Sigma}_l^{-1} + \Psi_i^{-1} + \rho^2\sigma_x^2\right)^{-2}\left(\frac{1}{1+2\rho^{-2}\sigma_x^{-2}\Psi_i^{-1}}\right)^2$$
$$-2\rho^{-2}\sigma_x^{-2}\left(\tilde{K} + \frac{1}{n}\sum_l \tilde{\Sigma}_l^{-1} + \Psi_i^{-1} + \rho^2\sigma_x^2\right)^{-1}\left(\frac{1}{1+2\rho^{-2}\sigma_x^{-2}\Psi_i^{-1}}\right)^3 < 0$$
$$\frac{\partial^2 \mathcal{L}}{\partial \tilde{\Sigma}_{wi}^{-1}\partial \tilde{\Sigma}_{wl}^{-1}} = 0 \qquad \text{for } i \neq l$$

Thus the Lagrangian is concave and the F.O.C. is necessary and sufficient for solution characterization. The solution to the problem is $\tilde{\Sigma}_{wj}^{-1} = K_2 + \frac{1}{n}\sum_i \tilde{\Sigma}_i^{-1} - \tilde{\Sigma}_j^{-1}$.

Appendix D. Proofs

Proof of Lemma 1.

Using the informational equivalent variable z_j I restate the manager's problem:

$$\max_{e\geq 0, m\geq 0} q_{1j}\left(e_j + N_j m_j\right) - \frac{\gamma}{2}q_{1j}^2 L_j - c_j\left(e_j, m_j\right)$$

The F.O.C. are necessary and sufficient for the solution characterization, delivering the result presented. □

Lemma 6. *Noisy traders expected loss is not affected by the managers' actions or contract. The contribution coming from asset j is given by:*

$$R_j = -\rho^{-1}\hat{\Sigma}_{aj}\tilde{\Sigma}_{wj}^{-1} + \rho\hat{\Sigma}_{aj}\left(\Psi_j^{-1} + V_{pj}^{-1}\right)\Psi_j\sigma_x^2 \tag{D.1}$$

Proof. Let R_j be the expected loss that noisy traders have from trading with informed traders (speculators) their stakes in payoff j. Letting p_{0j} be the value of the payoff j as of initial time, the equilibrium condition for noisy traders to hold shares of this payoff is given by the break-even equation:

$$r\left(1-\delta_j\right)p_{0j} = \left(1-\delta_j\right)\mathbb{E}_1\left[\pi_j - \left(h_{0j} + h_{1j}p_j\right)\right] - R_j \tag{D.2}$$

where δ_j is the stake held by insiders. The break-even equation illustrates that insiders ultimately bear the loss incurred by noisy traders upon trading with informed parties.

R_j must be equal to the ex-ante gain of speculators coming from trading the asset j, that is $R_j = \int_0^1 R_{ij}\,\mathrm{d}i$. Let i be an arbitrary speculator. His ex-ante expected gain $R_i = \mathbb{E}_{0i}\left[u_i^{*\prime}(\pi - \tilde{p})\right]$ is given by:

$$
\begin{aligned}
\mathbb{E}_{0i}\left[u_i^{*\prime}(\pi - \tilde{p})\right] = \rho^{-1} \Big\{ & (\beta e + \bar{\epsilon})'(I - A_1 - A_2 - B)'\hat{\Sigma}_i^{\prime -1} - s'\left(\tilde{\Sigma}^{-1} - \hat{\Sigma}_i^{-1}A_1\right)' - a_3'\hat{\Sigma}_i^{\prime -1} \\
& + m'\left[\tilde{\Sigma}^{-1} - \hat{\Sigma}_i^{-1}A_1 - \left(\tilde{\Sigma}_{\eta i}^{-1} + V_p^{-1} + \tilde{\Sigma}_w^{-1} - \hat{\Sigma}_i^{-1}(A_2 + B)\right)\alpha\right]'\Big\} \times \\
& \{(I - A_1 - A_2 - B)(\beta e + \bar{\epsilon}) - [A_1 + (I - A_2 - B)\alpha]\, m + A_1 s - a_3\} \\
& + \rho^{-1}\mathrm{trace}\left[(I - A_1 - A_2 - B)'\hat{\Sigma}_i^{\prime -1}(I - A_1 - A_2 - B)V_\chi\right] \\
& - \rho^{-1}\mathrm{trace}\left[\left(\tilde{\Sigma}^{-1} - \hat{\Sigma}_i^{-1}A_1\right)' A_1 \Sigma\right] \\
& + \rho^{-1}\mathrm{trace}\left[\left(\tilde{\Sigma}_{\eta i}^{-1} + V_p^{-1} + \tilde{\Sigma}_w^{-1} - \hat{\Sigma}_i^{-1}(A_2 + B)\right)'(I - A_2 - B)\Sigma_\epsilon\right] \\
& - \rho^{-1}\mathrm{trace}\left[\tilde{\Sigma}_w^{-1\prime} A_2 \tilde{\Sigma}_w\right] \\
& - \rho^{-1}\mathrm{trace}\left[C'\left(V_p^{-1}B^{-1} - \hat{\Sigma}_i^{-1}\right)' C\sigma_x^2 I\right]
\end{aligned}
$$

Integrating over i and substituting for the equilibrium pricing coefficients, I obtain the ex-ante expected gain of speculators:

$$
R = -\rho^{-1}\mathrm{trace}\left[\tilde{\Sigma}_w^{-1\prime}\hat{\Sigma}_a\right] + \rho\,\mathrm{trace}\left[\hat{\Sigma}_a\left(\Psi^{-1} + V_p^{-1}\right)\Psi\sigma_x^2 I\right]
$$

The contribution coming from asset j is given by:

$$
R_j = -\rho^{-1}\hat{\Sigma}_{aj}\tilde{\Sigma}_{wj}^{-1} + \rho\hat{\Sigma}_{aj}\left(\Psi_j^{-1} + V_{pj}^{-1}\right)\Psi_j\sigma_x^2
$$

which does not depend on the amount of effort or the report's bias or on the contract terms. □

Proof of Lemma 2.

The entrepreneur j's problem is to maximize his revenue V_j, that is, the revenue he receives from selling an arbitrary stake in the payoff to noisy traders plus the expected value of his holdings:

$$
\begin{aligned}
V_j &\equiv r\left(1-\delta_j\right)p_{0j} + \delta_j \mathbb{E}_0\left[\pi_j - \left(h_{0j} + h_{1j}p_j\right)\right] \\
&= \mathbb{E}_0\left[\pi_j\right] - R_j - \mathbb{E}_0\left[h_{0j} + h_{1j}p_j\right] \\
&= \mathbb{E}_0\left[\pi_j\right] - R_j - \frac{\gamma}{2}\mathbb{V}_0\left[T_j\right] - c\left(e_j, m_j\right) - \Omega
\end{aligned}
$$

where the first equality follows from the noisy traders break-even equation and the second equality follows from the manager's participation constraint.

By Lemma 6, the term R_j can be disregarded in the maximization. In maximizing V_j, the F.O.C. are necessary and sufficient for the solution characterization; they deliver the result presented.

□

Proof of Proposition 1.

(i) Calculating the derivatives:

$$
\frac{\mathrm{d}q_{1j}^*}{\mathrm{d}\gamma} = \frac{\partial q_{1j}^*}{\partial \gamma} + \frac{\partial q_{1j}^*}{\partial \kappa_j}\frac{\mathrm{d}\kappa_j}{\mathrm{d}\gamma}
$$

The first term is negative while the second is positive and depending on which prevails, the derivative can be positive or negative.

$$
\frac{\mathrm{d}m_j^*}{\mathrm{d}\gamma} = \frac{\partial m_j^*}{\partial q_{1j}^*}\frac{\mathrm{d}q_{1j}^*}{\mathrm{d}\gamma} + \frac{\partial m_j^*}{\partial \kappa_j}\frac{\mathrm{d}\kappa_j}{\mathrm{d}\gamma}
$$

The first term can be positive or negative while the second is negative. If $\frac{\mathrm{d}q_{1j}^*}{\mathrm{d}\gamma} < 0$, then $\frac{\mathrm{d}m_j^*}{\mathrm{d}\gamma} < 0$.

(ii) Calculating the derivatives delivers the results.

(iii) Calculating the derivatives:

$$\frac{\mathrm{d}q^*_{1j}\big/\beta_j}{\mathrm{d}\beta_j} = \frac{\partial q^*_{1j}\big/\beta_j}{\partial \beta_j} + \frac{\partial q^*_{1j}\big/\beta_j}{\partial L_j}\frac{\mathrm{d}L_j}{\mathrm{d}\beta_j} + \frac{\partial q^*_{1j}\big/\beta_j}{\partial N_j}\frac{\mathrm{d}N_j}{\mathrm{d}\beta_j} > 0$$

where the inequality follows because all the three terms are positive.
Write $m^*_j = \kappa_j^{-1}\left[\left(q^*_{1j}\big/\beta_j\right)\left(\beta_j N_j\right) - k_j g_j\right]$. Since $\beta_j N_j$ does not depend on β_j, the sign of $\frac{\mathrm{d}m^*_j}{\mathrm{d}\beta_j}$ is the same as the sign of $\frac{\mathrm{d}q^*_{1j}\big/\beta_j}{\mathrm{d}\beta_j}$ which is positive.

(iv) Calculating the derivatives:

$$\frac{\mathrm{d}q^*_{1j}}{\mathrm{d}V_{\chi_j}} = \frac{\partial q^*_{1j}}{\partial L_j}\frac{\mathrm{d}L_j}{\mathrm{d}V_{\chi_j}} < 0$$

$$\frac{\mathrm{d}m^*_j}{\mathrm{d}V_{\chi_j}} = \frac{\partial m^*_j}{\partial q^*_{1j}}\frac{\mathrm{d}q^*_{1j}}{\mathrm{d}V_{\chi_j}} < 0$$

$\square$

Proof of Proposition 2.

I start by noting that the share of information conveyed through feedback from prices relative to the private information share $\frac{\phi_{feed,j}}{\phi_{priv,j}} = \rho^{-2}\sigma_{xj}^{-2}\Psi_j^{-1}$ is decreasing in speculators' risk aversion and liquidity and increasing in the amount of private information.

(i) Let U_i denote speculator's i indirect utility and V_j entrepreneur's j value. Let $U \equiv \int_0^1 U_i \,\mathrm{d}i$ and $\sum_j V_j$ be their aggregates and let $S \equiv U + V$ denote the total welfare. Assume that $\mathrm{d}\phi_{state,j} = \mathrm{d}\phi_{publ,j} = 0$. Thus:

$$\mathrm{d}S = -\frac{\gamma}{2}q_{1j}^2\,\mathrm{d}L_j + \left(\frac{\rho^{-3}\sigma_{xj}^{-2}}{1 + 2\rho^{-2}\sigma_{xj}^{-2}\phi_{priv,j}\hat{\Sigma}_{aj}^{-1}} + \rho\sigma_{xj}^2\hat{\Sigma}_{aj}^2\right)\left(1 - \frac{\rho}{1 + \phi_{priv,j} + \rho^2\sigma_{xj}^2\hat{\Sigma}_{aj}^2}\right)\mathrm{d}\hat{\Sigma}_{aj}^{-1}$$

Because the shares of information conveyed through the financial statement and through public information are kept fixed, it is easy to check from eq. (1) that an increase in the amount of information diminishes the uncertainty faced by the manager under the contract. Thus it is welfare improving.

Furthermore, from eq. (3), I note that the contract performance sensitivity is decreasing in the uncertainty faced by the manager. Thus it increases as the amount of information increases. Finally, since the amount of information only affects the report's bias through the contract, I conclude that the report's bias increases.

(ii) and (iii) An increase either liquidity or speculators' risk aversion, by increasing the share of private information, increases the uncertainty faced by the manager under the contract as can be asserted from eq. (1). From eq. (3), I conclude that the contract performance sensitivity decreases. Since both liquidity and speculators' risk aversion impact on manipulation only through the contract, the report's bias decreases.

□

Proof of Proposition 3.

(i) Calculating the derivatives:

$$\frac{\mathrm{d}q_{1j}^*}{\mathrm{d}g_j} = \frac{\partial q_{1j}^*}{\partial \kappa_j}\frac{\mathrm{d}\kappa_j}{\mathrm{d}g_j} > 0$$

Write $m_j^* = \gamma^{-1}\left(1-k_j\right)^{-1}\left(k_j^{-1}N_j g_j^{-2} q_{1j}^* - g_j^{-1}\right)$.

$$\begin{aligned}\frac{\mathrm{d}m_j^*}{\mathrm{d}g_j} &= \gamma^{-1}\left(1-k_j\right)^{-1}\left[k_j^{-1}N_j\left(-2g_j^{-3}q_{1j}^* + g_j^{-2}\frac{\mathrm{d}q_{1j}^*}{\mathrm{d}g_j}\right) + g_j^{-2}\right] \\ &= \gamma^{-1}\left(1-k_j\right)^{-1}k_j^{-1}N_j g_j^{-3} q_{1j}^*\left(-2 + \frac{\mathrm{d}q_{1j}^*}{\mathrm{d}g_j}\frac{g_j}{q_{1j}^*} + \frac{k_j g_j}{q_{1j}^* N_j}\right)\end{aligned}$$

Since $N_j q_{1j}^* \geq k_j g_j$, it follows that:

If $\dfrac{\mathrm{d}q_{1j}^*}{\mathrm{d}g_j}\dfrac{g_j}{q_{1j}^*} < 1$, then $\dfrac{\mathrm{d}m_j^*}{\mathrm{d}g_j} < 0$.

If $\dfrac{\mathrm{d}q_{1j}^*}{\mathrm{d}g_j}\dfrac{g_j}{q_{1j}^*} > 2$, then $\dfrac{\mathrm{d}m_j^*}{\mathrm{d}g_j} > 0$.

(ii) Computing the derivative of the contract performance sensitivity with respect to the resource cost:

$$
\begin{aligned}
\frac{\mathrm{d}q_{1j}^*}{\mathrm{d}\alpha_j} &= \frac{\partial q_{1j}^*}{\partial \alpha_j} + \frac{\partial q_{1j}^*}{\partial N_j}\frac{\mathrm{d}N_j}{\mathrm{d}\alpha_j} \\
&= \frac{\kappa_j^{-1}\left[\beta_j^{-1}\left(1-\phi_{state,j}\right)\left(\alpha_j + 2N_j q_{1j}^*\right) - N_j\right]}{\gamma L_j + c_{ej}^{-1} + \kappa_j^{-1}N_j^2} \\
&= \frac{\kappa_j^{-1}\beta_j^{-1}\left[2\alpha_j + 2N_j q_{1j}^* - \phi_{state,j}\left(1 + 2\alpha_j + 2N_j q_{1j}^*\right)\right]}{\gamma L_j + c_{ej}^{-1} + \kappa_j^{-1}N_j^2}
\end{aligned}
$$

Let $\phi_j' \equiv \frac{2\alpha_j + 2k_j g_j}{1+2\alpha_j+2k_j g_j}$ and $\phi_j'' \equiv \frac{1+2\alpha_j}{2+2\alpha_j}$. If $\phi_{state,j} < \phi_j'$, then $\frac{\mathrm{d}q_{1j}^*}{\mathrm{d}\alpha_j} > 0$. If $\phi_{state,j} > \phi_j''$, then $\frac{\mathrm{d}q_{1j}^*}{\mathrm{d}\alpha_j} < 0$.

Computing the derivative of the report's bias with respect to the incremental resource cost:

$$
\frac{\mathrm{d}m_j^*}{\mathrm{d}\alpha_j} = \frac{\partial m_j^*}{\partial N_j}\frac{\mathrm{d}N_j^*}{\mathrm{d}\alpha_j} + \frac{\partial m_j^*}{\partial q_{1j}^*}\frac{\mathrm{d}q_{1j}^*}{\mathrm{d}\alpha_j}
$$

The ambiguity in $\frac{\mathrm{d}q_{1j}^*}{\mathrm{d}\alpha_j}$ renders $\frac{\mathrm{d}m_j^*}{\mathrm{d}\alpha_j}$ ambiguous; under the condition that guarantee $\frac{\mathrm{d}q_{1j}^*}{\mathrm{d}\alpha_j} < 0$ it follows that $\frac{\mathrm{d}m_j^*}{\mathrm{d}\alpha_j} < 0$.

□

Proof of Proposition 4. Consider the region at which $m_j > 0$, that is, region characterized by $k_j > 0$, $N_j > 0$ and $\underline{q}_j N \geq k_j g_j$. Also, $q_j = \underline{q}_j$ in this region. Computing the derivatives:

$$\frac{\mathrm{d}q_j}{\mathrm{d}\phi_{publ,j}} = \frac{\partial q_j}{\partial L_j}\frac{\mathrm{d}L_j}{\mathrm{d}\phi_{publ,j}} - \kappa_j^{-2}\frac{\partial q_j}{\partial \kappa_j^{-1}}\frac{\partial \kappa_j}{\partial k_j}\frac{\mathrm{d}k_j}{\mathrm{d}\phi_{publ,j}} > 0$$

$$\begin{aligned}\frac{\mathrm{d}m_j}{\mathrm{d}\phi_{publ,j}} &= \frac{\partial m_j}{\partial k_j}\frac{\mathrm{d}k_j}{\mathrm{d}\phi_{publ,j}} + \frac{\partial m_j}{\partial \kappa_j}\frac{\partial \kappa_j}{\partial k_j}\frac{\mathrm{d}k_j}{\mathrm{d}\phi_{publ,j}} + \frac{\partial m_j}{\partial q_{1j}}\frac{\partial q_{1j}}{\partial \kappa_j}\frac{\mathrm{d}\kappa_j}{\mathrm{d}k_j}\frac{\mathrm{d}k_j}{\mathrm{d}\phi_{publ,j}} \\ &\quad + \frac{\partial m_j}{\partial q_{1j}}\frac{\partial q_{1j}}{\partial L_j}\frac{\mathrm{d}L_j}{\mathrm{d}\phi_{publ,j}} \\ &= \kappa_j^{-2}N_j\gamma g_j^2\left(1-2k_j\right)q_{1j}^*\left(-1-\frac{k_j g_j}{N_j q_{1j}^*}\frac{k_j}{1-2k_j}+\frac{\partial q_{1j}^*}{\partial \kappa_j}\frac{\kappa_j}{q_{1j}^*}\right)\frac{\mathrm{d}k_j}{\mathrm{d}\phi_{publ,j}} \\ &\quad + \frac{\partial m_j}{\partial q_{1j}}\frac{\partial q_{1j}}{\partial L_j}\frac{\mathrm{d}L_j}{\mathrm{d}\phi_{publ,j}}\end{aligned}$$

The last term is positive and the first term is positive if $k_j < 1/3$ and $\frac{\partial q_{1j}^*}{\partial \kappa_j}\frac{\kappa_j}{q_{1j}^*} > 2$. The latter condition holds if $2\beta_j c_{ej}^{-1} < \kappa_j^{-1}N_j(3\alpha_j + k_j g_j)$, that is, for sufficiently low values of risk aversion for the manager.

Since the last term is bounded, even for values of capacity approaching zero or for values of manager's risk aversion approaching infinity, m_j is decreasing in scrutiny in economies where $\frac{\partial q_{1j}^*}{\partial \kappa_j}\frac{\kappa_j}{q_{1j}^*} < 1$ and sufficiently low values of $\phi_{publ,j}$. The former condition holds if $\kappa_j^{-1}N_j(2\alpha_j+1) < \beta_j c_{ej}^{-1}$, that is, for sufficiently high values of risk aversion for the manager, while $\frac{\mathrm{d}k_j}{\mathrm{d}\phi_{publ,j}}$ goes to infinity when either $\phi_{publ,j}$ goes to zero. □

Proof of Lemma 3.

Without loss of generality, take the detection probability as fixed. If the manager releases no public information under this assumption, he still would not do it when doing so increases the probability of having his manipulation detected. Moreover, this could even be thought as being the relevant case because the information would not be obtained as a result of scrutiny activity. Consider the value of payoff j's manager:

$$q_{1j}\left(e_j^* + N_j m_j^*\right) - \frac{\gamma}{2} q_{1j}^2 L_j - \frac{c_{ej}}{2} e_j^{*2} - k_j g_j m_j^* - \frac{\gamma}{2} k_j\left(1-k_j\right) g_j^2 m_j^{*2}$$

For a fixed contract, then m_j^* is either 0 or $\kappa_j^{-1}\left(q_{1j}N_j - k_j g_j\right)$. The total differential of the manager's value for the former and latter case respectively are given by:

$$\gamma q_{1j}^2 \beta_j^{-2} \Sigma_{\epsilon j}\, \mathrm{d}\phi_{state,j}$$

$$\left[\kappa_j^{-1}\left(q_{1j}N_j - k_j g_j\right) q_{1j}\beta_j^{-1}\left(1+\alpha_j\right) + \gamma q_{1j}^2 \beta_j^{-2}\Sigma_{\epsilon j}\right] \mathrm{d}\phi_{state,j}$$

In either case, the manager's value is decreasing in $\phi_{publ,j}$. □

Proof of Proposition 5.

Computing speculators' total differential (asset j information shares):

$$\mathrm{d}U = \left(1 + \phi_{priv,j} + \hat{\Sigma}_{aj}\rho^2\sigma_x^2\right)^{-1} \mathrm{d}\phi_{priv,j}$$

I consider different regions for the insiders' values.

Region I: $m_j > 0$, $q_j = \underline{q}_j$

In this region the envelope theorem holds. Also, it must be the case that $k_j > 0$, $N_j > 0$ and $\underline{q}_j N \geq k_j g_j$.

$$\begin{aligned} \mathrm{d}V = &\left[\left(\alpha_j\left(m^*\gamma g(1-2k_j)+1\right)\kappa_j^{-1} + \frac{\gamma}{2} g(1-2k_j) m_j^2 + \frac{1-2k_j}{1-k_j} m + g_j k_j \kappa_j^{-1}\right)\times\right. \\ &\left. g_j k_j' \hat{\Sigma}_{aj}^{-1} + \rho^{-1}\right] \mathrm{d}\phi_{publ,j} - \left(\frac{\gamma}{2} q_{1j}^2 \beta_j^{-2}\hat{\Sigma}_{aj} + \rho^{-1}\right)\mathrm{d}\phi_{priv,j} \\ &+ \left[\gamma q_{1j}^2\beta_j^{-2}\Sigma_{\epsilon j} - \left(\alpha_j\kappa_j^{-1} + m_j + g_j k_j \kappa_j^{-1}\right) q_{1j}\beta_j^{-1}\left(1+\alpha_j\right)\right]\mathrm{d}\phi_{state,j} \end{aligned}$$

Region II: $m_j = 0$, $q_j = N_j^{-1} k_j g_j$

In this region the envelope theorem does not hold. Also, $k_j > 0$, $N_j > 0$, $\underline{q}_j N_j < k_j g_j$ and $\bar{q}_j N_{\geq} k_j g_j$.

$$
\begin{aligned}
\mathrm{d}V = &\Big[\Big(\beta_j c_{ej}^{-1} g - \gamma g^2 L_j N^{-1} k - c_{ej}^{-1} g^2 N^{-1} k\Big) N^{-1} k' \hat{\Sigma}_{aj}^{-1} + \rho^{-1}\Big] \mathrm{d}\phi_{publ,j} \\
&+ \Big[\gamma g^2 N^{-2} k^2 \beta_j^{-2} \Sigma_{\epsilon j} - \Big(\beta_j c_{ej}^{-1} g - \gamma g^2 L_j N^{-1} k - c_{ej}^{-1} g^2 N^{-1} k\Big) \times \\
&kN^{-2} \beta_j^{-1}\Big(1+\alpha_j\Big)\Big] \mathrm{d}\phi_{state,j} - \Big(\frac{\gamma}{2} g^2 N^{-2} k^2 \beta_j^{-2} \hat{\Sigma}_{aj} + \rho^{-1}\Big) \mathrm{d}\phi_{priv,j}
\end{aligned}
$$

Region III: $m_j = 0$, $q_j = \bar{q}_j$

In this region the envelope theorem holds. Also, $N_j \leq 0$ always falls in this region. If $N_j > 0$, then it must be the case that $k_j > 0$, $\underline{q}_j N_j < k_j g_j$ and $\bar{q}_j N_j < k_j g_j$.

$$
\mathrm{d}V = \gamma q_{1j}^2 \beta_j^{-2} \Sigma_{\epsilon j} \, \mathrm{d}\phi_{state,j} - \left(\frac{\gamma}{2} q_{1j}^2 \beta_j^{-2} \hat{\Sigma}_{aj} + \rho^{-1}\right) \mathrm{d}\phi_{priv,j} + \rho^{-1} \, \mathrm{d}\phi_{publ,j}
$$

Collecting the appropriate terms corresponding to each case delivers the result.

□

Proof of Proposition 6.

$$
\begin{aligned}
\frac{\mathrm{d}\underline{q}_j}{\mathrm{d}\phi_{state,j}} = &\frac{-\alpha_j \kappa_j^{-1} \beta_j^{-1}(1+\alpha_j)}{\gamma L_j + c_{ej}^{-1} + \kappa_j^{-1} N_j^2} + \frac{\underline{q}_j}{\gamma L_j + c_{ej}^{-1} + \kappa_j^{-1} N_j^2} \quad \times \\
&\left\{\gamma \beta_j^{-2}\Big[\hat{\Sigma}_{aj}(1 + 2\rho^{-2} \sigma_{xj}^{-2} \hat{\Sigma}_{aj}^{-1} \phi_{priv,j})^{-1} + 2\Sigma_{\epsilon_j}\Big] - 2\kappa_j^{-1} N_j \beta_j^{-1}(1+\alpha_j)\right\}
\end{aligned}
$$

Define:

$$
\begin{aligned}
L_{1j} &\equiv \beta_j^{-2}\left[V_{\chi j} + \hat{\Sigma}_{aj} + \left(1 - \frac{2\alpha_j}{(1+\alpha_j)}\right)\Sigma_{\epsilon j} + \rho^2 \sigma_x^2 \hat{\Sigma}_{aj}^2\right] \\
L_{2j} &\equiv L_j(1 - \phi_{publ,j})
\end{aligned}
$$

$$\Delta_{1j}(c_{ej}) \equiv \beta_j^2 c_{ej}^{-2} - 4k_j g_j(\alpha_j + k_j g_j)\left[\kappa_j^{-1} - \frac{2\gamma\Sigma_{\epsilon j}}{\alpha_j(1+\alpha_j)}\right]c_{ej}^{-1} - 4\gamma k_j g_j(\alpha_j + k_j g_j)\kappa_j^{-1} L_{1j}$$
$$\Delta_{2j}(c_{ej}) \equiv \beta_j^2 c_{ej}^{-2} - 4k_j g_j(\alpha_j + k_j g_j)\kappa_j^{-1} c_{ej}^{-1} - 4\gamma k_j g_j(\alpha_j + k_j g_j)\kappa_j^{-1} L_{2j}$$

$$\mathcal{E}_{1j} \equiv \left\{c_{ej} \in \mathbb{R}_{++} : \Delta_{1j}(c_{ej}) \geq 0\right\}$$
$$\mathcal{E}_{2j} \equiv \left\{c_{ej} \in \mathbb{R}_{++} : \Delta_{2j}(c_{ej}) \geq 0\right\}$$
$$\mathcal{D}_j \equiv \left\{c_{ej} \in \mathbb{R}_{++} : c_{ej}^{-1} < \beta_j^{-2}\kappa_j^{-1}\alpha_j[1-(1+\alpha_j)\phi_{publ,j}]\right\}$$

$q_j^{ub} : \mathbb{R}_{++} \to \mathbb{R}_+$ such that:

$$q_j^{ub}(c_{ej}) = \begin{cases} \dfrac{\beta_j c_{ej}^{-1} + \Delta_{1j}(c_{ej})^{1/2}}{2\gamma L_{1j} + 2\left[1 - 2\alpha_j^{-1}(1+\alpha_j)^{-1}\gamma\kappa_j\Sigma_{\epsilon j}\right]c_{ej}^{-1}} & \text{for} \quad c_{ej} \in \mathcal{D}_j \cap \mathcal{E}_{1j} \\[2ex] \dfrac{\beta_j c_{ej}^{-1}}{\gamma L_{1j} + \left[1 - 2\alpha_j^{-1}(1+\alpha_j)^{-1}\gamma\kappa_j\Sigma_{\epsilon j}\right]c_{ej}^{-1}} & \text{for} \quad c_{ej} \in \mathcal{D}_j \cap \mathcal{E}_{1j}^c \\[2ex] \dfrac{\beta_j c_{ej}^{-1} + \Delta_{2j}(c_{ej})^{1/2}}{2\gamma L_{2j} + 2c_{ej}^{-1}} & \text{for} \quad c_{ej} \in \mathcal{D}_j^c \cap \mathcal{E}_{2j} \\[2ex] \dfrac{\beta_j c_{ej}^{-1}}{\gamma L_{2j} + c_{ej}^{-1}} & \text{for} \quad c_{ej} \in \mathcal{D}_j^c \cap \mathcal{E}_{2j}^c \end{cases}$$

By construction, for each $c_{ej} \in \mathbb{R}_{++}$:

$$q_j^{ub}(c_{ej}) \geq \sup_{\left\{\phi_{state,j} :\, \underline{q}_{1j} N_j \geq k_j g_j\right\}} \underline{q}_{1j}(\phi_{state,j} | c_{ej})$$

For $\underline{c}_j \leq c_{ej}^{-1} < \bar{c}_j$, $q_j^{ub}(c_{ej}) < \dfrac{\beta_j(1+\alpha_j)(\alpha_j + 2k_j g_j)}{\gamma\kappa_j\left(\hat{\Sigma}_{aj} + 2\Sigma_{\epsilon j}\right)}$ and the result that $\dfrac{\mathrm{d}\underline{q}_{1j}}{\mathrm{d}\phi_{state,j}} < 0$ holds.

Furthermore, if $\dfrac{\mathrm{d}\underline{q}_j}{\mathrm{d}\phi_{state,j}} < 0$ and $\gamma \leq \dfrac{(1+\alpha_j)\rho\sigma_{xj}}{k_j^{1/2}(1-k_j)^{1/2}g_j}$ then $\underline{q}_{1j}$ is concave in $\phi_{state,j}$, which implies that m_j^* is also concave in $\phi_{state,j}$.

□

Lemma 7. *Let ϕ_{0j} denote the value of $\phi_{state,j}$ for which $\underline{q}_j = 0$. The quantity $\underline{q}_j N_j$ is increasing in $\phi_{state,j}$ on $\left[\frac{\alpha_j}{1+\alpha_j}, \phi_{qN,j}\right]$ and decreasing on $[\phi_{qN,j}, \phi_{0j}]$, where $\phi_{qN,j}$ is the unique point at which $\frac{\mathrm{d}(\underline{q}_j N_j)}{\mathrm{d}\phi_{state,j}} = 0$.*

Proof. Observe that the value of $\phi_{state,j}$ for which $N_j = 0$ is given by $\frac{\alpha_j}{1+\alpha_j}$. Taking the first derivative of $\underline{q}_j N_j$ with respect to $\phi_{state,j}$:

$$\frac{\mathrm{d}(\underline{q}_j N_j)}{\mathrm{d}\phi_{state,j}} = \frac{c_{ej}^{-1}(1+\alpha_j) - \alpha_j \frac{\mathrm{d}(\kappa_j^{-1}N_j^2)}{\mathrm{d}\phi_{state,j}} - q_{1j}^* N_j \left(-2\gamma\beta_j^{-2}\Sigma_{\epsilon j} + \frac{\mathrm{d}(\kappa_j^{-1}N_j^2)}{\mathrm{d}\phi_{state,j}}\right)}{\gamma L_j + c_{ej}^{-1} + \kappa_j^{-1}N_j^2}$$

Let $Q_a \equiv c_{ej}^{-1}(1+\alpha_j) - \alpha_j \frac{\mathrm{d}(\kappa_j^{-1}N_j^2)}{\mathrm{d}\phi_{state,j}}$ and $Q_b \equiv -2\gamma\beta_j^{-2}\Sigma_{\epsilon j} + \frac{\mathrm{d}(\kappa_j^{-1}N_j^2)}{\mathrm{d}\phi_{state,j}}$. It follows from the fact that $\underline{q}_j N_j$ is zero at the endpoints of the interval $\left[\frac{\alpha_j}{1+\alpha_j}, \phi_{0j}\right]$ and positive in the interior that $Q_a\left(\frac{\alpha_j}{1+\alpha_j}\right) > 0$ and $Q_a(\phi_{0j}) < 0$. The derivative is zero at a given point if and only if $\underline{q}_j N_j = \frac{Q_a}{Q_b}$ at that point. Taking the second derivative:

$$\begin{aligned}\frac{\mathrm{d}^2(q_{1j}^* N_j)}{\mathrm{d}\phi_{state,j}^2} = &-\frac{\alpha_j + q_{1j}^* N_j}{\gamma L_j + c_{ej}^{-1} + \kappa_j^{-1}N_j^2}\frac{\mathrm{d}^2(\kappa_j^{-1}N_j^2)}{\mathrm{d}\phi_{state,j}^2} - 2\frac{-2\gamma\beta_j^{-2}\Sigma_{\epsilon j} + \frac{\mathrm{d}(\kappa_j^{-1}N_j^2)}{\mathrm{d}\phi_{state,j}}}{\gamma L_j + c_{ej}^{-1} + \kappa_j^{-1}N_j^2} \times \\ &\left[c_{ej}^{-1}(1+\alpha_j) - \alpha_j \frac{\mathrm{d}(\kappa_j^{-1}N_j^2)}{\mathrm{d}\phi_{state,j}} - q_{1j}^* N_j\left(-2\gamma\beta_j^{-2}\Sigma_{\epsilon j} + \frac{\mathrm{d}(\kappa_j^{-1}N_j^2)}{\mathrm{d}\phi_{state,j}}\right)\right] \end{aligned} \tag{D.3}$$

The second derivative is negative at all critical points of $q_{1j}^* N_j$ on the interval $\left[\frac{\alpha_j}{1+\alpha_j}, \phi_{0j}\right]$. So there exists only one local maximum (also a global maximum) on that interval. Let $\phi_{qN,j}$ denote that point. Thus $\frac{\mathrm{d}(q_{1j}^* N_j)}{\mathrm{d}\phi_{state,j}} > 0$ for any $\phi_{state,j} \in \left[\frac{\alpha_j}{1+\alpha_j}, \phi_{qN,j}\right)$ and $\frac{\mathrm{d}(q_{1j}^* N_j)}{\mathrm{d}\phi_{state,j}} < 0$ for any $\phi_{state,j} \in (\phi_{qN,j}, \phi_{0j}]$. □

Lemma 8. *Let* $\bar{\gamma}_j \equiv \left(\frac{\beta_j^2}{2\Sigma_{\epsilon j} g^2}\right)^{1/2} \left[\frac{\alpha_j N_j(\phi_{1j})}{g k_j(\phi_{1j})} \frac{\mathrm{d}k_j^{-1}(1-k_j)^{-1}N_j}{\mathrm{d}\phi_{state,j}}(\phi_{1j}) + \frac{\mathrm{d}k_j^{-1}(1-k_j)^{-1}N_j^2}{\mathrm{d}\phi_{state,j}}(\phi_{1j})\right]^{1/2}$*. If* $\gamma > \bar{\gamma}_j$ *, then* $\underline{q}_j$ *is increasing in* $\phi_{state,j}$ *on* $\left[\phi_{1j}, \phi_{qj}\right]$ *and decreasing on* $\left[\phi_{qj}, \phi_{0j}\right]$*, where* $\phi_{qj} < \phi_{qN,j}$ *is the unique point at which* $\frac{\mathrm{d}\underline{q}_j}{\mathrm{d}\phi_{state,j}} = 0$*; otherwise,* $\underline{q}_j$ *is decreasing in* $\phi_{state,j}$ *on* $\left[\phi_{1j}, \phi_{0j}\right]$.

Proof. Since $\underline{q}_j N_j$ is decreasing on $(\phi_{qN,j}, \phi_{0j}]$, then $\underline{q}_j$ is decreasing on this interval. In what follows, I establish the growth pattern of $\underline{q}_j$ on $[\phi_{1,j}, \phi_{qN,j}]$. Taking the first derivative of $\underline{q}_j$ with respect to $\phi_{state,j}$:

$$\frac{\mathrm{d}\underline{q}_j}{\mathrm{d}\phi_{state,j}} = \frac{-\alpha_j \frac{\mathrm{d}\kappa_j^{-1}N_j}{\mathrm{d}\phi_{state,j}} - \underline{q}_j\left(-2\gamma\beta_j^{-2}\Sigma_{\epsilon j} + \frac{\mathrm{d}\kappa_j^{-1}N_j^2}{\mathrm{d}\phi_{state,j}}\right)}{\gamma L_j + c_{ej}^{-1} + \kappa_j^{-1}N_j^2}$$

If $\gamma > \bar{\gamma}_j$, then $\frac{\mathrm{d}\underline{q}_j}{\mathrm{d}\phi_{state,j}}(\phi_{1j}) > 0$. By continuity, there exists $\phi_{qj} \in (\phi_{1j}, \phi_{qN,j})$ such that $\frac{\mathrm{d}\underline{q}_j}{\mathrm{d}\phi_{state,j}}(\phi_{qj}) = 0$ and $\frac{\mathrm{d}\underline{q}_j}{\mathrm{d}\phi_{state,j}} > 0$ on $[\phi_{1j}, \phi_{qj})$. At point ϕ_{qj}:

$$\underline{q}_j(\phi_{qj}) = \frac{\alpha_j \frac{\mathrm{d}(\kappa_j^{-1}N_j)}{\mathrm{d}\phi_{state,j}}(\phi_{qj})}{2\gamma\beta_j^{-2}\Sigma_{\epsilon j} - \frac{\mathrm{d}(\kappa_j^{-1}N_j^2)}{\mathrm{d}\phi_{state,j}}(\phi_{qj})}$$

Taking the second derivative of $\underline{q}_j$ with respect to $\phi_{state,j}$:

$$\frac{\mathrm{d}^2\underline{q}_j}{\mathrm{d}\phi_{state,j}^2} = -\frac{\alpha_j + \underline{q}_j}{\gamma L_j + c_{ej}^{-1} + \kappa_j^{-1}N_j^2} \frac{\mathrm{d}^2(\kappa_j^{-1}N_j)}{\mathrm{d}\phi_{state,j}^2} - 2\frac{-2\gamma\beta_j^{-2}\Sigma_{\epsilon j} + \frac{\mathrm{d}(\kappa_j^{-1}N_j)}{\mathrm{d}\phi_{state,j}}}{\gamma L_j + c_{ej}^{-1} + \kappa_j^{-1}N_j^2} \times \left[-\alpha_j \frac{\mathrm{d}(\kappa_j^{-1}N_j)}{\mathrm{d}\phi_{state,j}} - \underline{q}_j\left(-2\gamma\beta_j^{-2}\Sigma_{\epsilon j} + \frac{\mathrm{d}(\kappa_j^{-1}N_j)}{\mathrm{d}\phi_{state,j}}\right)\right] \tag{D.4}$$

All critical points of $\underline{q}_j$ are local maxima, which implies that ϕ_{qj} is the unique local maximum (also the global maximum) on the interval $(\phi_{1j}, \phi_{qN,j})$. Thus $\underline{q}_j$ is increasing on $\left[\phi_{1j}, \phi_{qj}\right]$ and decreasing on $\left[\phi_{qj}, \phi_{0j}\right]$. I still have to show that $\phi_{qj} \le \phi_{2j}$ and I will address this later on.

If $\gamma < \bar{\gamma}_j$, then $\frac{\mathrm{d}\underline{q}_j}{\mathrm{d}\phi_{state,j}}(\phi_{1j}) < 0$. In this case, $\underline{q}_j$ must be decreasing on the whole interval $\left[\phi_{1j}, \phi_{0j}\right]$. If that were not the case, there would exist at least a local maximum and a local minimum on the interval $(\phi_{1j}, \phi_{qN,j})$. However, the second derivative revealed that $\underline{q}_j$ has no local minimum on $(\phi_{1j}, \phi_{qN,j})$.

Finally, if $\gamma = \bar{\gamma}_j$, then $\frac{\mathrm{d}\underline{q}_j}{\mathrm{d}\phi_{state,j}}(\phi_{1j}) = 0$. From the second derivative, point ϕ_{1j} must necessarily be a local maximum of $\underline{q}_j$ on the interval $(\phi_{1j}-\tau, \phi_{1j}+\tau)$ for some arbitrary $\tau > 0$. This implies that $\underline{q}_j$ is decreasing on the whole interval $\left[\phi_{1j}, \phi_{0j}\right]$. Otherwise, there would exist at least a local maximum and a local minimum on the interval $(\phi_{1j}, \phi_{qN,j})$ which, as I have shown, cannot be the case. □

Lemma 9. *Let Φ_{mj} be the set of local extrema of m_j^* on $\left(\phi_{1j}, \phi_{2j}\right)$. This set is non-empty and its infimum is a local maximum and greater than $\phi_{qN,j}$.*

Proof. Because m_j^* is zero at the endpoints of the interval $[\phi_{1j}, \phi_{2j}]$, there exists at least one local maximum. Since m_j^* is continuous, there are at most countably many of them. Letting $\Phi_{mj} = \{\phi_{mj,1}, \phi_{mj,2}, \ldots\}$ denote the set of local extrema, with $\phi_{mj,1} < \phi_{mj,2} < \cdots$, it follows that $\phi_{mj,k}$ is a local maximum if k is odd and a local minimum if k is even.

Let $f_j \equiv \underline{q}_j N_j - k_j g_j$ and consider the derivative of m_j^* with respect to $\phi_{state,j}$:

$$\frac{\mathrm{d}m_j^*}{\mathrm{d}\phi_{state,j}} = \frac{\mathrm{d}\kappa_j^{-1}}{\mathrm{d}\phi_{state,j}} f_j + \kappa_j^{-1} \frac{\mathrm{d}f_j}{\mathrm{d}\phi_{state,j}}$$

For any point ϕ in Φ_{mj}:

$$f_j(\phi) = -\left(\frac{\mathrm{d}\kappa_j^{-1}}{\mathrm{d}\phi_{state,j}}(\phi)\right)^{-1} \kappa_j(\phi)^{-1} \frac{\mathrm{d}f_j}{\mathrm{d}\phi_{state,j}}(\phi)$$

Since $f_j(\phi) > 0$, then $\frac{\mathrm{d}f_j}{\mathrm{d}\phi_{state,j}}(\phi) < 0$, which requires $\frac{\mathrm{d}\underline{q}_j N_j}{\mathrm{d}\phi_{state,j}}(\phi) < 0$. Thus, by virtue of Lemma 7, $\phi_{qN,j} < \phi_{mj,1}$. □

Corollary 3. $\phi_{qj} < \phi_{mj,1} < \phi_{2j}$.

Corollary 4. *Let $\phi_{mj,0} \equiv \phi_{qj}$ if ϕ_{qj} exists and $\phi_{mj,0} \equiv \phi_{1j}$ otherwise. On any interval $(\phi_{mj,2k}, \phi_{mj,2k+1})$ with $k = 0, 1, 2, \ldots$, the report's bias m_j^* and the contract performance* *sensitivity q_j^* move in opposite directions, that is, m_j^* increases while q_j^* decreases. Moreover, while the report's bias is increasing on any of those intervals, detection probability is decreasing.*

Proof of Proposition 7. The result follows from Lemmas 7 to 9 and Corollaries 3 and 4. □

Appendix E. Learning technology

I have used a linear learning technology in the model, while other papers on portfolio choice under rational inattention literature opt for a learning technology based on an entropy measure instead (e.g., Peng and Xiong, 2006; Mondria, 2010; Van Nieuwerburgh and Veldkamp, 2010). Some of the reasons that make an entropy technology compelling are its scale neutrality and implied specialized learning in some settings (e.g. CRRA utility)(see Van Nieuwerburgh and Veldkamp, 2010).

I chose an additive constraint for two reasons. First, in the environment with CARA utility, the standard entropy learning technology produces the unappealing knife-edge result that speculators would be indifferent to any learning allocation. Secondly, even considering a perturbed version of the entropy technology that restores specialization, the implied aggregate information distribution, upon which my results depend, would be similar. I now show that, like the additive learning technology, a perturbed version of the entropy technology implies that there is more aggregate private information for assets whose financial statements are less precise.

Suppose that speculators must take a two-step decision. First, each speculator chooses a vortex of a regular n-sided polygon, where each vortex is associated to a payoff. I interpret the position as being a model counterpart to the fact that analysts tend to track assets associated to specific industries. Then, each speculator who chose vortex k would make an information choice subject to the following perturbed version of the entropy learning technology:

$$\frac{\prod_j \hat{\Sigma}_j^{-\epsilon_{kj}}}{\prod_j \tilde{\Sigma}_j^{-\epsilon_{kj}}} \leq \ln K$$

I interpret ϵ_{kj} as a measure of the difficulty of learning about payoff j for a speculator positioned at vortex k. Higher values of ϵ_{kj} correspond to greater learning difficulties. I assume that ϵ_{kj} depends on the distance between vortices k and j, the measure of speculators located at vortex j and the informativeness of the financial statement $\tilde{\Sigma}_j^{-1}$. This assumption conveys the idea that (i) if channels for uncovering information get crowded (greater measure of speculators learning about the payoff), the more difficult it becomes to learn about it, and (ii) the more the speculator knows about the payoff, as measured by the informativeness of the financial statement, the more difficult it is to uncover the residual uncertainty.

Paired with CARA utility, it implies learning specialization: each speculator located at vortex k learns about the asset j such that ϵ_{kj} is the lowest. In equilibrium, speculators located at vortex k learn solely about asset k and $\min\{e_{k1},\ldots,e_{kn}\} = \min\{e_{l1},\ldots,e_{ln}\}$, for all k and l. This implies that the measure of speculators must be greater for assets whose financial statement are less precise.

Appendix F. Contract linked to final payoff

In the class of the signal jamming models, the agent's horizon is usually crucial. Once the agent does not care about the price, the myopic behavior ceases to exist (see Stein, 1989). For this reason, the fact that the contract depends on stock price is essential to having manipulation in equilibrium. My results do not hinge on the restriction I placed on the contract though. I show that even if I allow for the contract to depend on the final payoff, the entrepreneur would still choose to provide incentives via prices. Suppose that the linear contract consists of a salary h_{0j}, stock appreciation rights h_{1j}, and a share of the final payoff h_{2j} that the entrepreneur transfers from his account to the manager's $(h_{2j} \leq \delta_j)$:

$$
\begin{aligned}
T_j &= h_{0j} + h_{1j}p_j + h_{2j}\Big[\pi_j - (h_{0j} + h_{1j}p_j)\Big] \\
&= h_{0j}(1-h_{2j})(1-\tilde{h}_{1j}) + \tilde{h}_{1j}(1-h_{2j})\tilde{p}_j + h_{2j}\pi_j
\end{aligned}
$$

where $\tilde{p}_j = h_{0j} + \left(1 + h_{1j}\right) r p_j$ and $\tilde{h}_{1j} \equiv \frac{h_{1j}}{(1+h_{1j})r}$.

Instead of price, using solely public information, I construct the normalized performance variable:

$$
\begin{aligned}
z_j &\equiv \frac{\tilde{p}_j + A_{1j}s_j - \left(A_{1j} + A_{2j} + B_j\right)\bar{\epsilon}_j}{\beta_j\left(A_{1j} + A_{2j} + B_j\right)} \\
&= e_j + \beta_j^{-1}\chi_j + \frac{A_{1j} - \left(A_{2j} + B_j\right)\alpha_j}{\beta_j\left(A_{1j} + A_{2j} + B_j\right)} m_j - \frac{A_{1j}}{\beta_j\left(A_{1j} + A_{2j} + B_j\right)}\xi_{\theta j} \\
&\quad + \frac{A_{2j}}{\beta_j\left(A_{1j} + A_{2j} + B_j\right)}\xi_{wj} + \frac{A_{2j} + B_j}{\beta_j\left(A_{1j} + A_{2j} + B_j\right)}\xi_{\epsilon j} + \frac{C_j}{\beta_j\left(A_{1j} + A_{2j} + B_j\right)} x
\end{aligned}
$$

The financial statement conveys pieces of information about effort not conveyed by the final payoff. Because the normalized performance variable constitutes a means for the entrepreneur to assess this information, it will be used in the contract. Unless, of course, the financial statement cannot be used as a predictor of the final payoff ($\Sigma_j = \infty$). In contrast, the final payoff will be used by the entrepreneur in the contract except in case speculators have unlimited learning capacity which drives the shares of information conveyed through financial statement, public information and private information shares down to zero and which also causes prices to fully reveal speculators' information ($C_j = 0$)[24]. Using the normalized performance variable z_j, I rewrite the contract:

$$
\begin{aligned}
T_j &= h_{0j}(1 - h_{2j})(1 - \tilde{h}_{1j}) + \tilde{h}_{1j}(1 - h_{2j})(-A_{1j}s_j + \bar{\epsilon}_j) + \tilde{h}_{1j}(1 - h_{2j})\beta_j z_j + h_{2j}\pi_j \\
&= q_{0j} + q_{1j}z_j + q_{2j}\pi_j
\end{aligned}
$$

where:

$$
\begin{aligned}
q_{0j} &= h_{0j}(1 - h_{2j})(1 - \tilde{h}_{1j}) + \tilde{h}_{1j}(1 - h_{2j})(-A_{1j}s_j + \bar{\epsilon}_j) \\
q_{1j} &= \tilde{h}_{1j}(1 - h_{2j}) \\
q_{2j} &= h_{2j}
\end{aligned}
$$

Defining the variables $\underline{q}_{1j}$ and $\underline{q}_{2j}$ as follows:

[24] I also note that if it were possible for the entrepreneur to condition the contract on the financial statement, he would still use the price (along with the financial statement) to provide incentives to the manager.

$$\underline{q}_{1j} = \frac{\beta_j\Big[(\beta_j^2 c_{ej}^{-1} - \alpha_j\kappa_j^{-1})\Sigma_{\epsilon j} - \alpha_j\kappa_j^{-1}\ (1+\alpha_j)V_{\chi j}\Big]\gamma\phi_{state,j}}{\Big[\Big(\kappa_j^{-1}\alpha_j(1+\alpha_j) + \gamma\Sigma_{\epsilon j}\Big)\phi_{state,j} - \kappa_j^{-1}\alpha_j^2 - \gamma\Big(V_{\chi j} + \Sigma_{\epsilon j}\Big) - c_{ej}^{-1}\beta_j^2\Big]^2}$$

$$\underline{q}_{2j} = \frac{\gamma(\beta_j^2 c_{ej}^{-1} + \alpha_j^2\kappa_j^{-1})[(1+\phi_{priv,j})\hat{\Sigma}_{aj} + \rho^2\sigma_{xj}^2\hat{\Sigma}_{aj}^2 - \phi_{state,j}\Sigma_{\epsilon j}]}{\Big[\Big(\kappa_j^{-1}\alpha_j(1+\alpha_j) + \gamma\Sigma_{\epsilon j}\Big)\phi_{state,j} - \kappa_j^{-1}\alpha_j^2 - \gamma\Big(V_{\chi j} + \Sigma_{\epsilon j}\Big) - c_{ej}^{-1}\beta_j^2\Big]^2} + \frac{\kappa_j^{-1}(1+\alpha_j)\phi_{state,j}[(\beta_j^2 c_{ej}^{-1}(1+\alpha_j) - \alpha_j\gamma\Sigma_{\epsilon j})\phi_{state,j} + \alpha_j\gamma(V_{\chi j} + \Sigma_{\epsilon j})]}{\Big[\Big(\kappa_j^{-1}\alpha_j(1+\alpha_j) + \gamma\Sigma_{\epsilon j}\Big)\phi_{state,j} - \kappa_j^{-1}\alpha_j^2 - \gamma\Big(V_{\chi j} + \Sigma_{\epsilon j}\Big) - c_{ej}^{-1}\beta_j^2\Big]^2}$$

and assuming that $\underline{q}_{1j}\beta_j^{-1}\Big((1-\alpha_j)\phi_{state,j} - \alpha_j\Big) - \alpha\underline{q}_{2j} \geq k_j g_j$ (existence of manipulation in equilibrium), the contract terms chosen[25] are $q_{1j}^* = \underline{q}_{1j}$ and $q_{2j}^* = \underline{q}_{2j}$. In this setting, the contract performance sensitivity is expressed by $q_{1j}^* + \beta_j q_{2j}^*$.

Appendix G. Pricing response

I have assumed throughout this paper that the liquidity shock hitting liquidity traders has mean zero, which implies that they are equally likely to increase or decrease their position in a given asset. This assumption leads to an incongruous result for the pricing response to manipulation detection[26].

Suppose that, after prices are observed and trading plans are submitted but before trading proceeds, the true state is revealed if manipulation is detected. Market participants observe possibly different prices, submit new plans and only then trades are carried out. In such circumstance, the impact on $\mathbb{E}[p]$ is null.

However, if the liquidity shock has a non-zero mean, that is, if liquidity traders on average do change their holdings, then the expected price changes by:

[25] I refrain from characterizing the contract for the case in which $\underline{q}_{1j}\beta_j^{-1}\Big((1-\alpha_j)\phi_{state,j} - \alpha_j\Big) - \alpha\underline{q}_{2j} < k_j g_j$, because the case considered suffices for the purpose of illustrating that the price is still used in the contract along with the final payoff.

[26] Empirical evidence indicates that, on average, stock returns fall about 10% on the days around which restatements are announced (Palmrose et al., 2004).

$\rho \bar{x}_j \left(\hat{\Sigma}_{aj,\text{before}} - \hat{\Sigma}_{aj,\text{after}} \right)$

where $\bar{x}_j$ denotes the liquidity shock mean.

Thus, the expected pricing response to manipulation detection is negative if and only if the average supply of asset coming from liquidity traders is negative ($\bar{x}_j < 0$)[27].

In order to understand why this is the case suppose that there is a positive net demand of asset coming from liquidity traders ($\bar{x} < 0$) and observe that manipulation detection causes speculators to be better informed about the payoff. Consider an extreme situation in which speculators are perfectly informed about the payoff of the asset. In such case, speculators position can only be bounded if price is equal to the payoff, which is also an indifference result as they would sell whatever is required to clear the market. Now, as their uncertainty increases, they would only sell the asset for a premium and thus the price would go up.

Moreover, the pricing response is decreasing in the share of information conveyed by the financial statement and in the average posterior precision. Certainly, the higher the share of information conveyed by the financial statement (or the higher the average posterior precision), the smaller is the uncertainty resolution brought about by manipulation detection and so the smaller is the impact on the premium required by speculators in order to meet the average demand of liquidity traders.

Finally, the liquidity shock mean, being proportional to $(1 - \delta)$, is positively correlated to liquidity. Thus I conclude that the pricing response is larger for more liquid claims.

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[27] Goldman and Slezak (2006) show a similar result for the pricing response by appealing to the presence of "naïve" market participants, who systematically underestimate the extent of the report's bias. I have shown that even when no market participant systematically misprices the asset, it is still possible to have that pricing response provided that liquidity traders on average increase their holdings.

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